HOMEWORK 6 **DUE: Mon., May 7** **NAME:**

DIRECTIONS:

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

You will lose point 0.5 points for each instruction not followed.

Questions	Points	Score
1	2	
2	2	
3	1	
4	2	
5	2	
6	1	
Total	10	

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Problem 1: (2 points) Let $x \in \mathbb{R}$, Show that there exists a sequence s such that $s_k \in \mathbb{Q}$ for all $k \in \mathbb{N}$ and $\lim_{n\to\infty} s_k = x$.

Problem 2: (2 points) Show that for any a, $b \in \mathbb{Q}$, we have $||a| - |b|| \leq |a - b|$.

Problem 3: (1 point) Show that the sum of two Cauchy sequence in $\mathbb{Q}$ is a Cauchy sequence in $\mathbb{Q}$.

Problem 4: (2 points) Show that addition is well defined in $\mathbb{R}$.

Problem 5: (2 points) Let $(a_k)_{k\in\mathbb{N}}$ be a Cauchy sequence of rational numbers such that $(a_k)_{k\in\mathbb{N}} \notin \mathcal{I}$. Define the inverse sequence, $(b_k)_{k\in\mathbb{N}}$, by

$$b_k = \begin{cases} 1, & \text{for } k \leq N, \\ 1/a_k, & \text{for } k > N, \end{cases}$$

where for $n > N$ we know there exists an $r \in \mathbb{Q}^+$ such that $|a_k| > r$. Prove that $(b_k)_{k\in\mathbb{N}}$ is Cauchy.

Problem 6: (1 point) Prove that every bounded sequence in $\mathbb{R}$ has a convergent subsequence.