**HOMEWORK 9** **DUE: Wed., May 30** **NAME:**

**DIRECTIONS:**

- Turn in your homework as **SINGLE-SIDED** typed or handwritten pages.
- **STAPLE** your homework together. Do not use paper clips, folds, etc.
- **STAPLE** this page to the front of your homework.
- Be sure to write your name on your homework.
- Show all work, **clearly and in order**.

**You will lose point 0.5 points for each instruction not followed.**

| Questions | Points | Score |
|---|---|---|
| 1 | 1 | |
| 2 | 1 | |
| 3 | 2 | |
| 4 | 0.5 | |
| 5 | 0.5 | |
| 6 | 1 | |
| 7 | 1 | |
| 8 | 1 | |
| 9 | 1 | |
| 10 | 1 | |
| Extra Credit | 1 | |
| Total | 10 | |

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**Problem 1:** (1 point) Let $V$ and $W$ be vector spaces over a field $F$ and $T : V \to W$ a linear transformation.

**(a)** (0.5 points) Show that $T(\mathbf{0}_V) = \mathbf{0}_W$, where $\mathbf{0}_V$ and $\mathbf{0}_W$ are the additive identities of $V$ and $W$ respectively. *Hint: Use the definition of the additive identity.*

**(b)** (0.5 points) Show that $T(-\mathbf{v}) = -T(\mathbf{v})$, where $-\mathbf{v}$ is the additive inverse of $\mathbf{v}$ in $V$ and $-T(\mathbf{v})$ is the additive inverse of $T(\mathbf{v})$ in $W$. *Note, you CANNOT simply say that $T(-\mathbf{v}) = -T(\mathbf{v})$ by the definition of linear transformation because we do not know that '-1' is an element of our field $F$.*

**Problem 2:** (1 point) Let $V$ and $W$ be vector spaces over a field $F$ and $T : V \to W$ a linear transformation. Show that $T(V)$ is a subspace of $W$.

**Problem 3:** (2 points) Let $V$ and $W$ be linearly isomorphic, finite dimensional vector spaces over $F$. Prove

**(a)** (0.5 point) $T^{-1} : W \to V$ is a linear transformation.

**(b)** (0.5 point) $\dim V = \dim W$.

**(c)** (1 point) if $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$ is a basis for $V$, then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), ..., T(\mathbf{v}_n)\}$ is a basis for W.

**Problem 4:** (0.5 points) Let $V$ be a vector space over the field $F$. If $R, S, T \in \mathcal{L}(V)$, Prove that $R \circ (S + T) = (R \circ S) + (R \circ T)$

**Problem 5:** (0.5 points) Let $V$ be a vector space over the field $F$. Let $I \in \mathcal{L}(V)$ be defined by $I(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v} \in V$. Show that $T \circ I = I \circ T = T$ for all $T \in \mathcal{L}(V)$.

**Problem 6:** (1 point) Let $\mathbb{R}[x]$ be the vector space of polynomial functions in one variable over $\mathbb{R}$. Define the multiplication of polynomials in the usual way. Show that $\mathbb{R}[x]$ is a commutative algebra with identity.

**Problem 7:** (1 point) Show that the number of elements in $S_n$ is $n!$.

**Problem 8:** (1 point) Show that the composition of two elements of $S_n$ is also an element of $S_n$.

**Problem 9:** (1 point) Calculate the sign of all elements of $S_3$ using definition 2.4.4.

**Problem 10:** (1 point) Show that any $\sigma \in S_n$ can be decomposed into the composition of transpositions.

**Extra Credit 1:** (1 point) Show that $\sigma \in S_n$ is an odd permutation if and only if it is the composition of an odd number of transpositions. *(You may assume without proof that a transposition is an odd permutation.)*