

**DIRECTIONS:**

- No papers, phones, calculators, or gadgets are permitted to be out during the quiz.
- Show all work, clearly and in order **You will lose points if any of these instructions are not followed.**

Questions	Points	Score
1	1.5	
2	1.5	
3	2	
Total	5	

**Problem 1:** (1.5 point) Let  $A_i$  be sets for  $i = 1, 2, 3, \dots, n$ . What is the definition of the n-fold Cartesian Product of sets  $A_1, A_2, \dots, A_n$ ?

The n-fold Cartesian product of sets  $A_1, \dots, A_n$  is given by

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \dots, a_n) | a_j \in A_j \text{ for } j = 1, 2, \dots, n\}.$$

**Problem 2:** (1.5 point) State the Well-Ordering Principle for  $\mathbb{Z}$ .

If  $A$  is a non-empty subset of the positive integers, then  $A$  has a least element. That is, there exists an element  $a_0 \in A$  such that for all  $a \in A$ ,  $a_0 < a$ .

**Problem 3:** (2 points) Consider the set of integers,  $\mathbb{Z}$ . Prove that the multiplicative identity is unique.

**Proof:** Suppose 1 and  $1'$  are both multiplicative identities. Therefore, for all  $a \in \mathbb{Z}$

$$(1) \quad 1 \cdot a = a \cdot 1 = a,$$

and

$$(2) \quad 1' \cdot a = a \cdot 1' = a.$$

In particular (1) holds for  $a = 1'$  and (2) holds for  $a = 1$  hence

$$1 = 1 \cdot 1' = 1' \cdot 1 = 1'.$$

Therefore the multiplicative identity must be unique.