

**DIRECTIONS:**

- No papers, phones, calculators, or gadgets are permitted to be out during the quiz.
- Show all work, clearly and in order **You will lose points if any of these instructions are not followed.**

Questions	Points	Score
1	1	
2	2	
3	2	
Total	5	

**Problem 1:** (1 point) What is the definition of a finite set?

A set is said to be finite if there exists a bijection between that set and  $\mathbb{N}_k$  for some  $k \in \mathbb{N}$ .

**Problem 2:** (2 points) If  $A$  is a finite set and  $A \approx B$ , prove that  $B$  is finite and  $Card(A) = Card(B)$ .

**Proof:**  $A$  is finite means that  $A \approx \mathbb{N}_k$  and there exists a bijection  $f : \mathbb{N}_k \rightarrow A$ . But since  $A \approx B$  that means there exists a bijection  $g : A \rightarrow B$ . So  $g \circ f : \mathbb{N}_k \rightarrow B$  is also a bijection so  $B \approx \mathbb{N}_k$  and  $Card(B) = k$ .

Q.E.D.

**Problem 3:** (2 points) Label the following true or false

(a) (0.5 points) F  $\mathbb{N}$  is finite.

(b) (0.5 points) T  $A \approx \emptyset \iff A = \emptyset$ .

(c) (0.5 points) F If  $A$  and  $B$  are countable, then  $A \times B$  is equinumerous to either  $A$  or  $B$ .

(d) (0.5 points) T  $\mathbb{N} \approx \mathbb{N} \cup \{0\}$ .