

DIRECTIONS:

- No papers, phones, calculators, or gadgets are permitted to be out during the quiz.
- Show all work, clearly and in order **You will lose points if any of these instructions are not followed.**

Questions	Points	Score
1	1	
2	2	
3	2	
Total	5	

Problem 1: (1 point) State the Bolzano-Weierstrass Theorem.

Let S be a bounded, infinite subset of \mathbb{R} . Then S has an accumulation point.

Problem 2: (2 points) Let $a, b \in \mathbb{R}$ such that $a < b$. Prove that (a, b) is an open set.

Proof: Let $x \in (a, b)$, then by definition $a < x < b$. Take $\epsilon = \min\{x - a, b - x\}/2$. Then $(x - \epsilon, x + \epsilon) \subseteq (a, b)$.

Q.E.D.

Problem 3: (2 points) Label the following true or false

(a) (0.5 points) T A sequence of real numbers $(a_k)_{k \in \mathbb{N}}$ of real numbers is convergent if and only if it is Cauchy.

(b) (0.5 points) F Every monotonic increasing sequence in \mathbb{R} converges to an element in \mathbb{R} .

(c) (0.5 points) F Let $S = \mathbb{R} \setminus \mathbb{Q}$ be the set of irrational numbers. Then the set of all accumulation points of S is the set of rational numbers, \mathbb{Q} .

(d) (0.5 points) T Let $S \subseteq \mathbb{R}$. Then $x \in \mathbb{R}$ is an accumulation point of S if and only if every neighborhood of x contains infinitely many points of S .