QUIZ 7 **Wed., May 16** **NAME:**

DIRECTIONS:

- No papers, phones, calculators, or gadgets are permitted to be out during the quiz.
- Show all work, clearly and in order **You will lose points if any of these instructions are not followed.**

Questions	Points	Score
1	1	
2	2	
2	2	
Total	5	

Problem 1: (1 points) State the Exchange Lemma.

Let V be a vector space over a field F and suppose $U = \{\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_m\}$ is a spanning set. If $V = \{\mathbf{v}_1, \mathbf{v_2}, ..., \mathbf{v}_n\}$ is a linearly independent set, then $n \leq m$.

Problem 2: (2 points) Let V be a vector space over a field, F and let $\mathbf{v} \in V$ be a nonzero vector. Show that $\{\mathbf{v}\}$ is a linearly independent set. You may assume $\beta\mathbf{0} = \mathbf{0}$ for all $\beta \in F$.

Proof: Suppose that $\{\mathbf{v}\}$ was a linearly dependent set. Then there exists some $\alpha \in F$ with $\alpha \neq 0$ such that $\alpha\mathbf{v} = \mathbf{0}$. But F is a field so the multiplicative inverse $\alpha^{-1} \in F$ exists. Hence $\mathbf{v} = \alpha^{-1}\mathbf{v} = \mathbf{0}$ which is a contradiction.

Q.E.D.

Problem 3: (2 points) Label the following true or false

(a) (0.5 points) ___T___ $V = \mathbb{R}$ over $F = \mathbb{R}$ is a vector space.

(b) (0.5 points) ___F___ If a set of vectors is linearly dependent, then one of them must be the zero vector.

(c) (0.5 points) ___T___ All bases are spanning sets.

(d) (0.5 points) ___F___ All spanning sets are bases.