QUIZ 8 **WED., MAY. 23** **NAME:**

DIRECTIONS:

- No papers, phones, calculators, or gadgets are permitted to be out during the quiz.
- Show all work, clearly and in order **You will lose points if any of these instructions are not followed.**

Questions	Points	Score
1	1	
2	2	
2	2	
Total	5	

Problem 1: (1 points) State the definition of a subspace.

Let V be a vector space over a field, F, and $W \subseteq V$ a nonempty subset. Then we call W a subspace if it is closed under vector addition and scalar multiplication.

Problem 2: (2 points) Let V be a vector space over a field F and $S, T \in \mathcal{L}(V)$. Prove that the composition, $S \circ T \in \mathcal{L}(V)$.

Proof: Let $\mathbf{u}, \mathbf{v} \in V$ and $\alpha \in F$. Then $S \circ T(\mathbf{u}+\mathbf{v}) = S\left(T(\mathbf{u}+\mathbf{v})\right) = S\left(T(\mathbf{u})\right) + S\left(T(\mathbf{v})\right) = S \circ T(\mathbf{u}) + S \circ T(\mathbf{v})$. Similarly $S \circ T(\alpha\mathbf{v}) = S\left(T(\alpha\mathbf{v})\right) = S\left(\alpha T(\mathbf{v})\right) = \alpha S\left(T(\mathbf{v})\right) = \alpha S \circ T(\mathbf{v})$. Q. E. D.

Problem 3: (2 points) Label the following true or false

(a) (0.5 points) ___T___ $\mathbb{Q}[x]$ is an infinite dimensional vector field.

(b) (0.5 points) ___F___ The dimension of a vector space, V, is the same as the cardinality of V.

(c) (0.5 points) ___F___ All subsets of a vector space are themselves, vector spaces.

(d) (0.5 points) ___F___ Let V be an n-dimensional vector space. Then all bases of V have cardinality strictly less than n by the Exchange Lemma.