LIFE ON A PLANET ORBITING THE *ALPHA CENTAURI* STAR SYSTEM: A MATHEMATICAL MODEL

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Abstract. This paper proposes a mathematical model for the conditions necessary to sustain life on a planet orbiting the binary star system Alpha Centauri. The conditions investigated and discussed include the orbit radius and the existence of an atmosphere. In this paper, we shall prove that in order to sustain life on a planet orbiting Alpha Centauri, an atmosphere must be present. We also intend to calculate the habitable zone (range of orbit radii) of the planet. This model will potentially aid scientists in the search for extraterrestrial life on planets in the Alpha Centauri system as well as on planets in other star systems.

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1. Statement of Problem

We propose a model of the criteria necessary for a supposed planet, orbiting the binary star system Alpha Centauri, to be able to sustain carbon-based life. In order for the planet to sustain life similar to life found on Earth, liquid water must be present; liquid water is essential in order to sustain carbon-based life because many of the necessary biochemical reactions require water as a solvent [9]. Therefore, this planet would need to orbit its star with a radius that places it within a constrained radii range, which is known as a habitable zone. In this zone, thermal equilibrium is maintained which ensures the constant presence of liquid water [8]. The thermal aspect of the model is discussed in detail in Section 2.2. Furthermore, life on

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the planet would need to be protected from harmful high-energy radiation. This aspect of the model is discussed in Section 2.1. The mathematical model presented here has real world applications in the scientific community. This model will aid scientists in the search for extraterrestrial life on planets in the Alpha Centauri star system as well as on planets in other star systems by excluding planets that cannot sustain life. Furthermore, a life sustaining planet orbiting Alpha Centauri, which is the second closet star system to Earth, could potentially be a future destination for human colonization [7].

2. Model Design

2.1. X-Ray Analysis.

In order for the planet to sustain carbon-based life, the amount of high-energy radiation hitting the planet, such as x-ray radiation, must be kept to a minimum.[1] The amount of high-energy radiation striking the planet can be kept to a minimum in one of two ways: (1) The planet's orbit radius is large enough that the amount of radiation reaching the planet is negligible or (2) The planet has an atmosphere to protect the surface from harmful radiation.

2.1.1. *Option 1.* Let us first consider the possibility of option 1. X-ray emission can be kept to a life-sustaining minimum if the orbit radius is made sufficiently large. In order to prove that the radius lies outside the habitable zone, we need to know the amount of x-ray radiation carbon-based life forms can withstand. It is assumed that this minimum amount is approximately equal to the amount of x-ray radiation exposure on Earth's surface (where carbon-based life forms are able to exist). The average amount of cosmic ray exposure on Earth at ground level is approximately 2.0 mSv [4]. Using this value along with x-ray emission data for the star system Alpha Centauri, we can prove that the orbital radius needed to keep the x-ray emission to a life-sustaining minimum lies outside the habitable zone. The acquisition and analysis of the x-ray emission data is discussed in Appendix A.

2.1.2. *Option 2.* In the case that the orbit radius from option 1 lies outside the habitable zone, the planet must instead have an atmosphere to protect its surface from harmful high-energy radiation. In order to have an atmosphere similar to that of Earth, the planet in question must be terrestrial and have a mass of approximately 0.5 to 10 Earth masses [6]. This is due to the fact that the atmosphere of a terrestrial planet mainly consists of carbon dioxide and nitrogen gases, while the atmosphere of a Jovian (gas) planet is composed mostly of helium and hydrogen gases.

Our calculations in Section 3.4 prove that an atmosphere is necessary to protect life on the planet from high energy radiation.

2.2. Thermal Analysis.

2.2.1. *Assumptions.* Since we need an atmosphere and since planet size is fixed, this model assumes that a planet that can sustain life will have conditions very similar to those of Earth. Additionally, liquid water must be present for life to exist. Therefore, the temperature on the planet must be between 0 and 100°C [8]. An additional assumption is that the energy of a star radiates out uniformly in all

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[1] The following analysis is based solely on x-ray radiation. The results obtained using x-ray data are representative of the results that would be obtained through the analysis of all high-energy radiation.

directions, which allows us to precisely calculate the energy that a planet would receive. Another assumption is that the orbit of planet is uniformly circular; even though most planets have an elliptical orbit around their star, if a planet's orbit is highly eccentric, it would not be able to support life. An eccentric orbit would cause the temperature to continually fluctuate, resulting in temperatures outside of the required range (0 – 100°C). Also, all of the energy radiated out of star travels uninterruptedly. Finally, we assume that the power output of all stars other than the one the planet is orbiting is negligible because of the large distance.

Our model considers a planet only orbiting a single star. As the distance between Alpha-Centauri A and B is very large, a life-sustaining planet could not orbit both stars and receive enough energy. This assumption is justified in Section 3.3. Additionally, a planet orbiting both stars would be unstable [10].

2.2.2. *Mathematical Preliminaries.* We assume that the power output of the Sun is $3.827 \times 10^{26} W$ [3]. The minimum distance from Earth to the Sun is Sun is $1.46 \times 10^{11} m$ and the maximum distance is $1.52 \times 10^{11} m$[2]. The radius of the Earth is $6.378 \times 10^{6} m$, and it is negligible because it is five orders of magnitude smaller than the distance from the Earth to the Sun.

If we calculate the surface area of the sphere with a radius equal to the distance from the Earth to the Sun, for the minimum radius, we get

$$\text{Surface Area} = 4\pi r^2 = 4\pi(1.496 \times 10^{11})^2 = 2.68 \times 10^{23} m^2. \tag{2.1}$$

For the maximum radius, the surface area is $2.90 \times 10^{23} m^2$.

We are concerned with the power per unit area because that is the amount of energy that the planet would receive per square meter around its equator. Thus, for the maximum radius of orbit,

$$\frac{\text{Power}}{\text{Area}} = \frac{\text{Power Output of Sun}}{\text{Surface Area of Sphere}} = \frac{3.827 \times 10^{26} W}{2.90 \times 10^{23} m^2} = 1320 W/m^2, \tag{2.2}$$

and we define this constant as K_1. For the minimum radius of orbit,

$$\frac{\text{Power}}{\text{Area}} = \frac{\text{Power Output of Sun}}{\text{Surface Area of Sphere}} = \frac{3.827 \times 10^{26} W}{2.68 \times 10^{23} m^2} = 1430 W/m^2, \tag{2.3}$$

and we define this constant as K_2.

Since $K_n = P/(4\pi d_n^2)$ where P is the power output of the star, d is the distance from the star to the planet, and $n = 1, 2$, solving for d yields

$$d_n = \sqrt{\frac{P}{4\pi K_n}}\ . \tag{2.4}$$

Thus, the largest orbital radius radius necessary for a planet to sustain life is d_1 and the smallest radius is d_2.

2.2.3. *Elliptical Orbits.* Since we have two orbital radii for each planet, it is possible to fit an ellipse between these circles. The equation for an ellipse is

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \tag{2.5}$$

where a is the semimajor axis (half of the distance across the ellipse along its longest axis) and b is the semiminor axis (half of the distance across the ellipse along its shortest axis). Thus $a = d_1$ and $b = d_2$ where d_1 is the largest orbital radius d_2 is the smallest radius. An important fact is that given this equation for an ellipse, no points on the ellipse are inside or outside d_1 or d_2.

Additionally, a and b can any be numbers such that $d_2 \leq b \leq a \leq d_1$ as then the ellipse would lie in the shaded region in Figure 1.

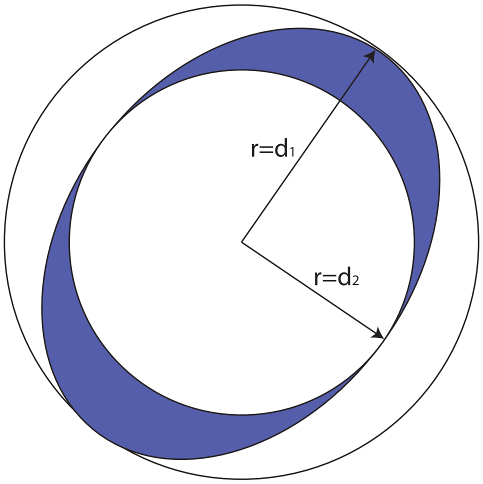

FIGURE 1. All possible elliptical orbits lay within the shaded area. The border of the shaded area is the ellipse with $a = d_1$ and $b = d_2$.

3. Model Implementation

3.1. Alpha Centauri A.

To find the orbital radii we use Equation 2.4. The power output of Alpha Centauri A is $5.813 \times 10^{26} W$ [3].

The maximum radius of orbit, d_{1A}, is

$$d_{1A} = \sqrt{\frac{5.813 \times 10^{26}}{4\pi K_1}} = 1.872 \times 10^{11} m \tag{3.1}$$

where $K_1 = 1430 W/m^2$.

The minimum radius of orbit, d_{2A}, is

$$d_{1A} = \sqrt{\frac{5.813 \times 10^{26}}{4\pi K_2}} = 1.799 \times 10^{11} m \tag{3.2}$$

where $K_2 = 1320 W/m^2$

Thus, the equation for the most elliptical orbit is

$$1 = \frac{x^2}{(1.872 \times 10^{11})^2} + \frac{y^2}{(1.799 \times 10^{11})^2}. \tag{3.3}$$

3.2. Alpha Centauri B.

The power output of Alpha Centauri B is $1.914 \times 10^{26} W$ [3].

The maximum radius of orbit, d_{1B}, is

$$d_{1A} = \sqrt{\frac{1.914 \times 10^{26}}{4\pi K_1}} = 1.074 \times 10^{11} m \tag{3.4}$$

where $K_1 = 1430 W/m^2$.

The minimum radius of orbit, d_{2B}, is

$$d_{1A} = \sqrt{\frac{1.914 \times 10^{26}}{4\pi K_2}} = 1.032 \times 10^{11} m \tag{3.5}$$

where $K_2 = 1320 W/m^2$

Thus, the equation for the most elliptical orbit is

$$1 = \frac{x^2}{(1.074 \times 10^{11})^2} + \frac{y^2}{(1.032 \times 10^{11})^2}. \tag{3.6}$$

3.3. **Potential for Dual Star Orbit.** The smallest distance between Alpha Centauri A and B is $1.67 \times 10^{12} m$, so if the planet were in the direct center of the two stars and orbiting them both in a figure-eight, it would be $8.35 \times 10^{11} m$ away from each star.

We can calculate the power per square meter received by the planet from each star. The surface area of a sphere with a radius of $8.35 \times 10^{11} m$ is $8.76 \times 10^{24} m^2$. The planet thus receives from Alpha Centauri A is

$$\frac{\text{Power}}{\text{Area}} = \frac{\text{Power Output of Alpha Centauri A}}{\text{Surface Area of Sphere}} = \frac{5.813 \times 10^{26} W}{8.76 \times 10^{24} m^2} = 67.06 W/m^2, \tag{3.7}$$

and the power received from Alpha Centauri B is $21.85 W/m^2$. The total power received per unit area of the planet is the sum of these two values, and equals $88.91 W/m^2$. This value is $1231 W/m^2$ smaller than K_1, so the planet would not receive enough power to orbit both planets.

3.4. **High Energy Radiation.** The average human exposure to radiation is $2.0 mSv$ per year, which corresponds to $0.16 J/yr$.

We averaged the amount of radiation detected by the Chandra satellite and converted the exposure times to the amount of energy it would collect over a year, assuming the emissions are constant over time. We found this value to be $0.021 J/yr$.

3.4.1. *Alpha Centauri A.* The product of the ratio of the emissions collected by the satellite to the area of the detector and the ratio of the surface area of the sphere of radius equal to the distance from Earth to Alpha Centauri to the surface area of the sphere generated with radius equal to the average radius of orbit for our hypothetical planet (d_{1A} and d_{2A} from Section 3.1 above) is equivalent to the total energy emitted by the star per meter squared:

$$\frac{\text{High Energy Radiation}}{\text{Meter}} = \left(\frac{0.21 J/yr}{0.006 m^2}\right)\left(\frac{4\pi(4.13x10^16)^2 m^2}{4\pi(1.835x10^11)^2 m^2}\right) = 1.77x10^11 J/m^2, \tag{3.8}$$

which is the total energy emitted by the star that reaches a square meter region on the equator of a planet orbiting Alpha Centauri A.

3.4.2. *Alpha Centauri B.* The calculations for the amount of energy hitting a planet orbiting Alpha Centauri B are identical to those described in Section 3.4.1, except the distance used to calculate the surface area of the sphere corresponding to the planet's orbit is the average of the maximum and minimum radii (d_{1B} and d_{2B} from section 3.2).

The total energy emitted by the star that reaches a square meter region on the equator of a planet orbiting Alpha Centauri B is $5.39x10^11J/m^2$.

Since this value for a planet orbiting either star is much greater than the amount of radiation absorbed by humans on Earth, it is clear that an atmosphere would be necessary to protect life on our theoretical planet.

4. Discussion

Our model proposes criteria necessary for a planet orbiting the Alpha Centauri system to sustain carbon-based life. Section 2.1 discussed the need for an atmosphere to protect against harmful high-energy radiation. Section 2.2 discussed the thermal energy requirements for liquid water, the existence of which is necessary for carbon-based life. For both of these sections, we used the Earth-Sun star system as a guide because it successfully supports carbon-based life. Combining these calculations we came up with a potential elliptical orbit for such a planet between the stars Alpha Centauri A and B. A planet in this elliptical orbit satisfies both the thermal and atmospheric requirements for sustaining life as we know it. For the calculations in Section 2.2, the potential orbit of such a planet is different for the A and B stars. For Alpha Centauri A, the range of orbit radii is $1.799 \times 10^{11}m$ to $1.872 \times 10^{11}m$. For Alpha Centauri B, the range of orbit radii is $1.032 \times 10^{11}m$ to $1.074 \times 10^{11}m$. An orbit around both planets is not possible because the planet would not receive enough power to support life.

Using the Earth Sun system as a guide exposes our model to weaknesses that could undermine the results outlined above. First, it is possible that extraterrestrial life may not be carbon-based and could therefore operate entirely differently than anything with which humans have experience. The existence of such a life form may not require the thermal requirements put forth in this report. A non-carbon based life form may not require liquid water, which is what the thermal analysis sought to ensure. Second, the atmospheric composition similar to that of Earth may not be the only way to protect against harmful radiation. By using the Earth-Sun system we assumed that the atmospheric composition would be similar to that of Earth to protect life as we know it. It is possible, however, that an alternate composition is possible that would drastically change the thermal analysis. An alternate composition would alter the heat retention of the planet in a way that would prevent it from behaving like Earth.

The model proposed in this report could be made more plausible by incorporating the possibility of non-carbon based life and atmospheric conditions dissimilar to that of Earth. Also the possible existence of life more adapted to exist in harsher conditions than those found on Earth would important to consider.

5. Conclusion

Based on the results of our analysis of x-ray radiation, our hypothetical planet orbiting one of the stars in the Alpha Centauri system would need an atmosphere to protect the surface from the high-energy radiation emitting from the star. This hypothetical planet could support carbon-based life if it also orbits within a calculated range of radii where a relatively stable temperature would maintain liquid water. We demonstrate that our single body model is fitting because any orbit around both stars would set the planet outside of our calculated habitable zone. We also show that a more likely elliptical planetary orbit would be possible within this habitable zone. Therefore, we can conclude from our initial conditions that a hypothetical planet orbiting either Alpha Centauri A or B could be able to support carbon-based life.

Appendix A. X-ray analysis

The x-ray data of the binary system is collected using the x-ray satellite Chandra. This data is available to the public via NASA Internet archives [5]. This data comes in the form of photon counts per energy level and can be analyzed with the HEAsoft software package provided by NASA. For this study, observation 7432 and 29 are used. The exposure times of these observations are 118220 seconds and 800200 seconds respectively. The sufficient length of these exposures ensures the accuracy and dependability of the data.

The data collected from the binary star system Alpha Centauri is analyzed using SAOimage DS9, an astronomical imaging and data visualization application. First, the binary system is selected and filtered using the DS9 image. The DS9 application is depicted in Figure 2. Then, from the selected region, an energy spectrum is extracted. The energy spectrum plots the distribution of photons and their energies that are emitted by the system within the x-ray range. Then, a basic step function model is fit to the extracted energy spectrum. After a model is fit to the data, the area under the model function is calculated. This integral yields the total x-ray energy emitted by the binary star system detected by Chandra.

However, the Chandra satellite only detects a fraction of the total energy emitted. This concept is depicted in Figure 3. Assuming that the binary star system emits

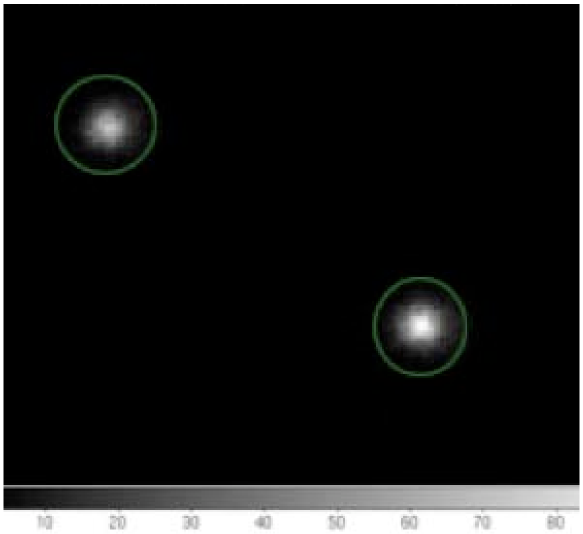

Figure 2. A depiction of the SAOimage DS9 window. This application is used to select and filter the x-ray data. The DS9 application allows the user to view an image of the source being analyzed. The dashed lines represent orbits, the solid sphere represents the total energy emission from the binary star system, and the wedges represent fractions of the total energy emissions.

energy radially, the emission striking the planet per square meter at the equator can be calculated using the following equation:

$$\frac{E_p}{m^2} = \left(\frac{E_s}{A}\right)\left(\frac{4\pi r_1^2}{4\pi r_2^2}\right) \tag{A.1}$$

where E_p is the x-ray emission striking the surface of the planet, E_s is the x-ray emission detected by the satellite, A is the detector collecting area of the satellite (which equals $0.006m^2$)[1], E_s is the x-ray energy emitted detected by the satellite, E_t is the total x-ray energy emitted by the star system, r_1 is the distance between the binary star system and the earth[2], r_2 is the difference between the star (either Alpha Centauri A or Alpha Centauri B) and the planet.

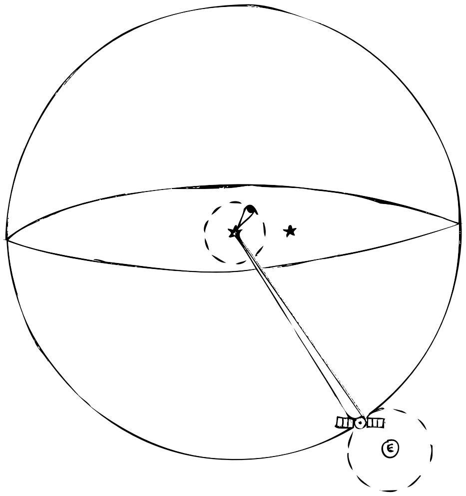

Figure 3. The amount of x-ray energy detected by Chandra is only a fraction of the total energy emitted by the binary star system.

References

[1] Chandra x-ray observatory. http://heasarc.gsfc.nasa.gov/docs/chandra/chandra.html.

[2] Earth statistics. http://www.windows.ucar.edu/tour/link=/earth/statistics.html.

[3] Pierre Kervella and Frederic Thevenin. ESO - eso0307 - a family portrait of the alpha centauri system. http://www.eso.org/public/news/eso0307/, March 2003.

[4] NASA. Understanding space radiation, October 2002.

[5] Phil Newman. HEASARC browse: Main interface. http://heasarc.gsfc.nasa.gov/cgi-bin/W3Browse/w3browse.pl.

[6] Prabhat. Difference between terrestrial and jovian planets | difference between. http://www.differencebetween.net/science/difference-between-terrestrial-and-jovian-planets/.

[7] Staffan Soderhjelm. Visual binary orbits and masses POST HIPPARCOS. *Astronomy and Astrophysics*, 341:121–140, 1999.

[8] Rich Townsend. mad star - the conditions for life. http://www.astro.wisc.edu/~townsend/static.php?ref=diploma-3#toc-The_Habitable_Zone, October 2009.

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[2] The distance between the satellite and Earth is negligable.

[9] Peter Tyson. NOVA | origins | life's little essential | PBS. http://www.pbs.org/wgbh/nova/origins/essential.html.

[10] Paul A. Wiegert and Matt J. Holman. The stability of planets in the alpha centauri system. *The Astronomical Journal*, 113:1445, 1997.