Winter 2010
Math 200 Group Project Preliminary Report
Group Members: Bailey Steinworth; Xing Zhang
Instructor: Dr. Eva Strawbridge
Date due: Feb 5 2010

ZOMBIFICATION OF THE PLANET

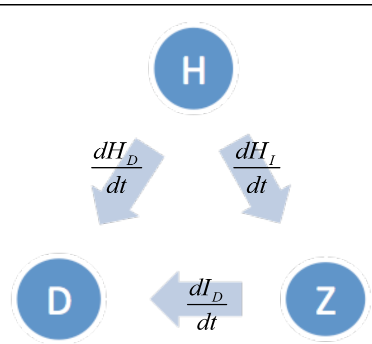
Abstract

Zombies, though are not proven to exist in real life (yet), remain to be a part of folklores and continue to haunt the dreams of many. Our innate fear of the undead is best epitomized in popular blockbusters like the *Resident Evil* film series. These films portray a "zombie apocalypse" in which the world is overwhelmed with zombie infection to such an extent that nearly all human institutions collapse and there remains only a handful of survivors who stay in small pockets, spend most of their time looking for supplies and face death/infection literally "around the corner" every second. In short, a zombie outbreak equals end of civilization and the human race itself. If this situation is indeed likely to happen, it will be a serious hidden threat to us. Thus our project aims to use mathematical modeling in the form of differential equations to find a pattern of the variation of the human and zombie populations, and then predict the limit of human population decline.

Problem Statement

We will start with a simple scenario. Suppose that at time t_0(we will take one day as the unit time), a zombifying virus leaks out of a research facility and infects one person. Every day, each zombie infects anyone it comes into contact with, and a certain proportion of those infected were killed by a group attack. In addition, some zombies are destroyed by the humans and a certain proportion of zombies are destroyed in accidents like falling off tall places or by natural elements like lightning, fire or flood). Will the human population get wiped out eventually?

Our study of zombie infection is, in principle, analogous to the study of epidemics in general. One general model of infectious diseases is the SIR model (Norman T. J. 1975), in which a given population is divided into three groups, namely those who are susceptible to infection (S), the infected population (I), and those who have recovered from the infection (R). We can illustrate our problem using a similar concept, as shown in the following diagram:



H: Uninfected human population at time t
Z: Active zombie population at time t
D: Total number of traumatic deaths/destructions (both humans and zombies)
$\frac{dH_I}{dt}$: Rate of infection
$\frac{dZ_D}{dt}$: Rate of destruction of zombies
$\frac{dH_D}{dt}$: Rate of traumatic death of humans

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Our purpose for this preliminary report is to relate $\frac{dH}{dt}$ and $\frac{dZ}{dt}$ to H and Z respectively in the form of a vector field and determine if there are any steady state (i.e. $\frac{dZ}{dt} = \frac{dH}{dt} = 0$) at some particular values of H and Z.

Model Design

Before we build our model, we will state some assumptions in order to simplify things:

1. In the absence of zombies, H increases geometrically (i.e. a constant population percentage growth represented by the constant G)
2. The only way of infection is through a zombie attack;
3. The zombies are totally mindless and will attack any live human they see. Age, gender or any other human characteristics do not influence the likelihood of being attacked by a zombie;
4. At all times, both humans and zombies exist in homogeneous populations, and both of them are free to redistribute themselves throughout the planet. In other words, the population density of both humans and zombies stay uniform at all times.
5. No vaccine against the zombifying virus is available and hence any live human is susceptible to the infection;
6. Within our time frame, zombies are considered indefinitely animate by themselves and can only perish in freak accidents or human destruction.

To begin modeling, suppose that the outbreak begins with one spontaneously appearing zombie and that each zombie is able to bite five people a day, turning them into zombies. The number of zombies,*Z*, with respect to time after the beginning of the outbreak in days, *t,* can be modeled by the equation

$$Z(t) = 6^{t-1},$$

where *Z*(0)=1. Assuming that the human population begins at 6.7 billion people, the number of humans, *H*, with respect to time in days, *t,* will be represented by the equation

$$H(t) = 6.7 \times 10^9 - 6^{t-1},$$

since each new zombie created represents one human lost. To find how long it will take for the entire human population to become zombies, set *H*(*t*)=0 and solve for *t,* which gives us *t*=13.6 days. Rounding this result, we see that the entire human population will have become zombies by the end of 14 days.

Before we begin to despair at this grim prognosis, we must consider that this simple model does not take into account many factors that will affect the human and zombie populations. This model assumes that the zombie population grows continuously and never decreases. In the event of an actual zombie outbreak, it is most likely that will undergo destruction accidentally.

In the light of the above assumptions, we can construct the differential equations

$$\frac{dH}{dt} = GH - (C_I - C_{DH})HZ \quad (1), \text{ and}$$

$$\frac{dZ}{dt} = (C_I - C_K)HZ - C_{DZ}Z \quad (2),$$

in which G, C_I, C_{DH}, and C_K and C_{DZ} are the rate constants for natural human population growth, rate of infection, human traumatic death zombie destruction involving humans, and zombie destruction that does not involve humans, respectively.

The world population as of 2009 is about 6.7 billion, or 6.7×10^9, and the population percentage growth of that year is 1.1%.* We will take these values and set $H(t=0) = 6.7 \times 10^9$ and $G = 0.011$.

In order to display our model in the form of a vector field, we have to set some values for the rate constants.

Let's assume that at t=0, the zombie infects 10 person and kills one each day. Then we have

$$C_I HZ = 10 \quad (4) \text{ and}$$

$$C_{DH} HZ = 1 \quad (5).$$

Taking Z(t=0) to be 1, we have

$C_I = 1.49 \times 10^{-9}$, and

$C_{DH} = 1.49 \times 10^{-10}$.

We then assume that each day, 1 in 100 zombies gets destroyed in freak accidents, being the mindless beings as they are. This implies that $C_{DZ} = 0.01$.

We further assume that initially, the humans kill 1 in 10 zombies. This gives us the equation

$C_K H(t=0)Z = 1$

when Z=10. Solving for C_K, we get

$C_K = 1.49 \times 10^{-10}$.

Using the values of the constants found above, we derive that

$$\frac{dH}{dt} = 0.011H - 1.64\times10^{-9}HZ \qquad (6) \text{ and}$$

$$\frac{dZ}{dt} = 1.34\times10^{-9}HZ - 0.01Z \qquad (7).$$

Using the software Wolfram Mathematica, We can then construct the vector field as shown in Plot 1. The function used here is the VectorPlot function which generates a vector plot of the field [f(x), f(y)] as a function of (x, y)[1].

[1] See Reference 4

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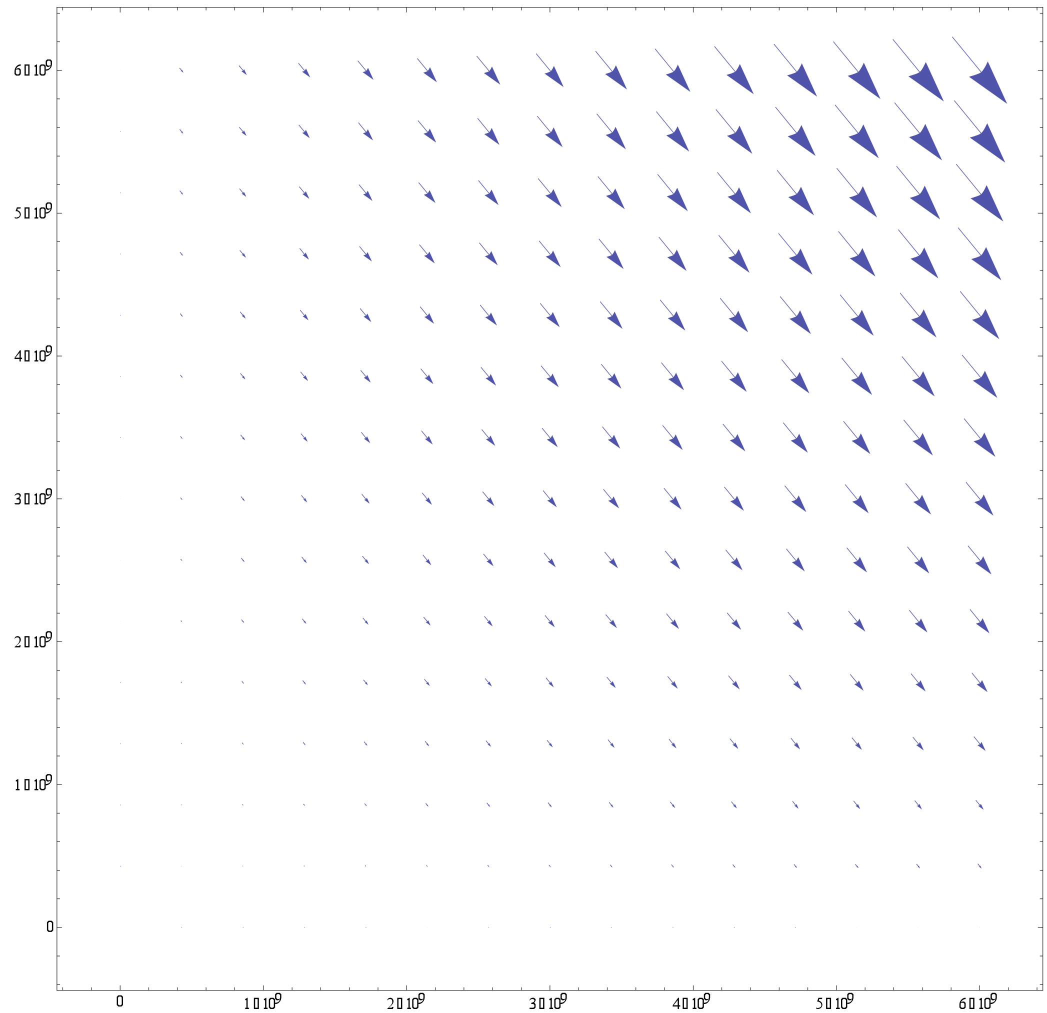
Plot 1



X-axis: Z
Y-axis: H

Vector equation: $\left(\frac{dZ}{dt}, \frac{dH}{dt}\right)=f(X,Y)=(1.34\text{x}10^{-9}\ XY - 0.01X,\ 0.011Y - 1.64 \text{ x } 10^{-9}\ XY)$

From Plot 1, we can see that the vectors are pointing to the positive X direction and negative Y direction, suggesting that for most part of the vector field, H is declining and Z is increasing. We also notice that for the same value of Z, $\frac{dZ}{dt}$ decreases in magnitude as H decreases. However, we are not yet able to see any apparent steady state points where the vectors converge in direction and approach the magnitude of 0. Therefore we will try to find a possible steady state point using the analytical method.

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If a steady state exists at a point (Z_s, H_s), then

$$\frac{dH_s}{dt} = 0.011H_s - 1.64\times10^{-9} H_s Z_s = 0 \qquad (8) \text{ and}$$

$$\frac{dZ_s}{dt} = 1.34\times10^{-9} H_s Z_s - 0.01Z_s = 0 \qquad (9)$$

at the point (Z_s, H_s).

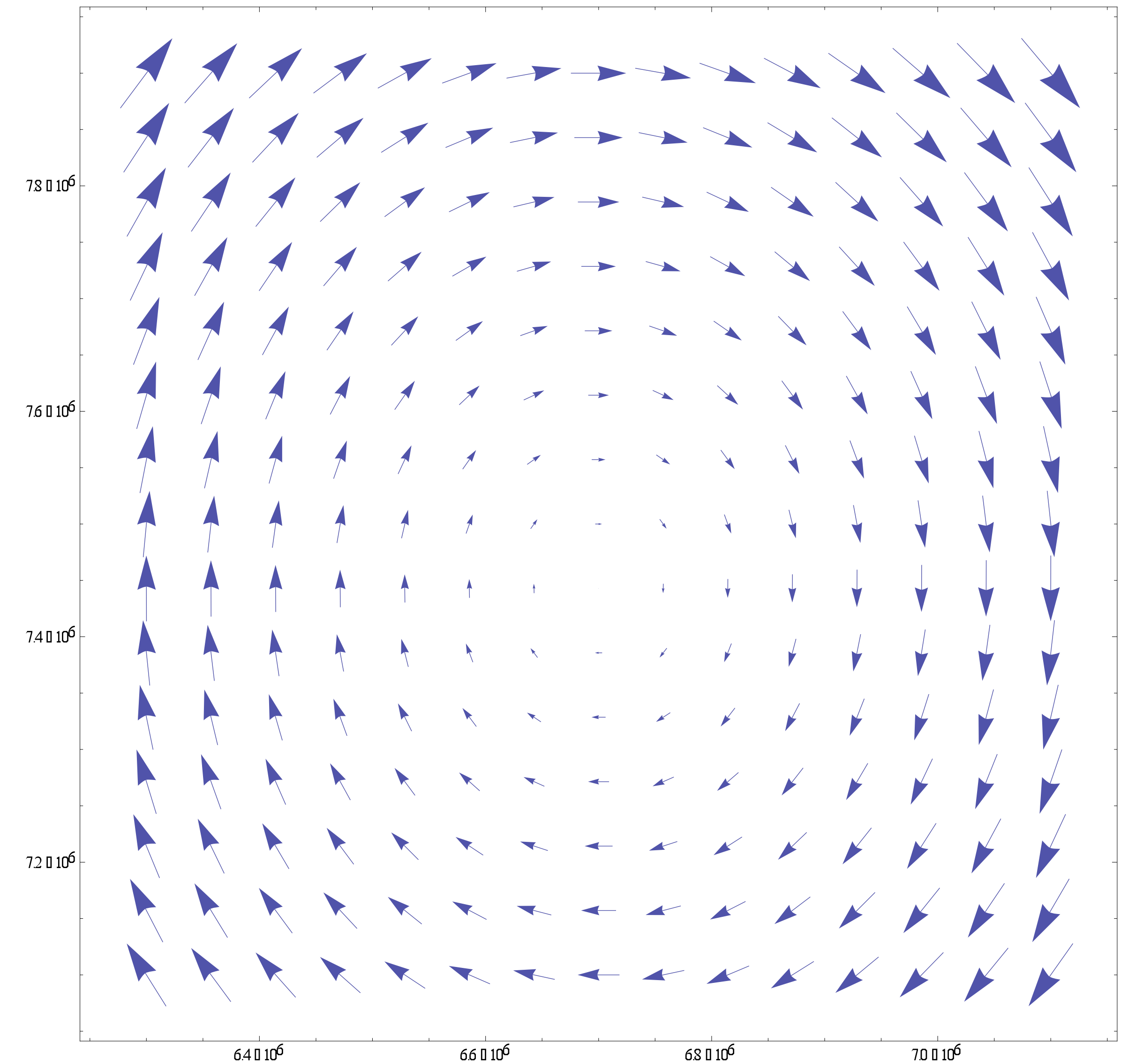
Solving for Z_s and H_s, we have

(Z_s, H_s) = (0, 0) (10) and

(Z_s, H_s) = ($6.7\text{x}10^6$, $7.5\text{x}10^6$). (11)

In this report, we are more interested in the point in Equation 11 since it suggests a steady state when neither H nor Z is 0. To confirm that steady state exists in this point, we study the portion of the vector field around this point in Plot 2 which is the region in Plot 1 immediately around the point in Equation 11.

Plot 2



X-axis: Z, range: $6.3\text{x}10^6 - 7.1\text{x}10^6$
Y-axis: H, range: $7.1\text{x}10^6 - 7.9\text{x}10^6$

Vector equation: $\left(\frac{dZ}{dt}, \frac{dH}{dt}\right)=f(X,Y)=(1.34\text{x}10^{-9}\,XY - 0.01X,\ 0.011Y - 1.64 \text{ x } 10^{-9}\,XY)$

Plot 2 shows that as we approach the point ($6.7\text{x}10^6$, $7.5\text{x}10^6$), the vectors' magnitudes decrease and approach 0. What is interesting about this point is that at another point close to it, the direction of the vector appears to be perpendicular to the line joining that point to the steady state point. This implies that a slight displacement form this steady state point can result in a situation in which H and Z increase and decrease periodically. To confirm this, we make use of Mathematica's StreamPlot function[2] to plot the paths of the human and zombie populations.

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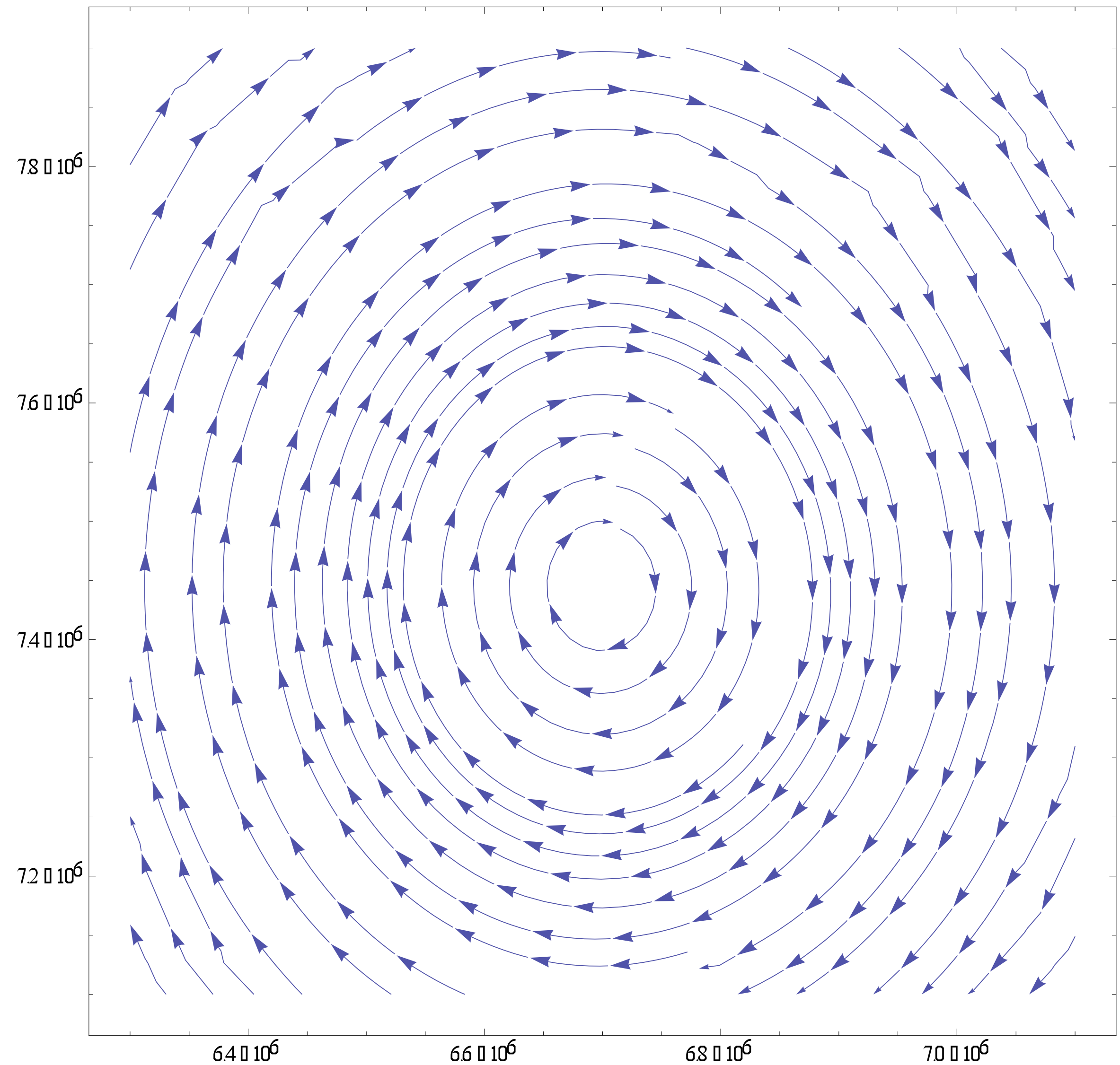
[2] The stream function ψ can be expressed as:

$$\mathbf{u} = \nabla \times \psi$$

Where u is the velocity vector (in our case u = $(\frac{dZ}{dt}, \frac{dH}{dt})$).

(See Reference 5)

Plot 3



X-axis: Z, range: $6.3\text{x}10^6 - 7.1\text{x}10^6$

Y-axis: H, range: $7.1\text{x}10^6 - 7.9\text{x}10^6$

Vector equation: $(\frac{dZ}{dt}, \frac{dH}{dt})$=f(X,Y)=($1.34\text{x}10^{-9}$ XY – 0.01X, 0.011Y-$1.64 \text{ x } 10^{-9}$ XY)

In plot 3, we can see that the paths indeed form concentric, closed circles around the steady state point ($6.7\text{x}10^6$, $7.5\text{x}10^6$). Thus this steady state point is a neutral steady point at which if the H/Z ratio is slightly displaced, the ratio is moving neither towards nor away from the steady state, but rather fluctuates periodically. In this sense, the human/zombie populations have reached a dynamic equilibrium.

Conclusion

Viewing the vector and stream plots, we can see that near the point (6.7×10^6, 7.5×10^6), the populations oscillate and remain in a dynamic equilibrium. There will be periodic population fluctuations, but ultimately the populations will not vary greatly.

In conclusion, it appears that in the event of a zombie outbreak, it will be possible to maintain a human population. There are still additional questions, though, about whether the human and zombie populations will reach this maintainable point.

As long as there are exactly 7.5×10^6 humans and 6.7×10^6 zombies, both populations will remain stable, ensuring the survival of humanity. However, we still do not know if the populations will ever reach this value. Plotting the number of zombies on the x-axis and the number of humans on the y-axis, we begin with t=0 at the point (1, 6.7×10^9). Following the vector flow line from this point, we would like to know whether the population will actually reach the steady-state point (6.7×10^6, 7.5×10^6). If it does reach this point, we would like to know how long it will take.

If the populations do not reach exactly (6.7×10^6, 7.5×10^6), they will enter a dynamic equilibrium in which they will fluctuate periodically this point. We would like to know what the maximum and minimum human and zombie populations will be in this cycle and how long each cycle will take.

We also hope to find what the maximum and minimum values for maintaining dynamic equilibrium are. Is there a range outside of which one or the other population will go to zero? If there is such a range, we plan to find what it is. Finding such a range will tell us what human population is absolutely necessary to maintain any human survival and if there is a zombie population below which we can entirely wipe out the undead.

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References

1. Bailey, Norman T. J. (1975). *The mathematical theory of infectious diseases and its applications* (2nd ed.). London: Griffin.

2. Sonia Altizer; Nunn, Charles (2006). *Infectious diseases in primates: behavior, ecology and evolution*. Oxford University Press.

3. U.S. Census Bureau, January 2010 (http://www.census.gov/ipc/www/idb/worldpop.php)

4. VectorPlot Function, Virtual Book (instruction manual), Wolfram Mathematica v.7.0.1.0, ©copyright 1988-2009 Wolfram Research Inc.

5. B. S. Massey and J. Ward-Smith, *Mechanics of Fluids*, 7th ed., Nelson Thornes, UK (1998).