Modeling of a Fish Population and the Effects of Overfishing

MATH 201 GROUP PROJECT FINAL REPORT

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Math 201 Group Project Final Report
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Abstract

According to the United Nations, 200 million people depend on fishing as their source of food or livelihood.[1] However, current fishing practices threaten the security of marine populations. Dramatic examples such as the collapse of the cod industry in Newfoundland in the early 1990s illustrate that fishing without appropriate limitations can have detrimental effects on fish populations and on the people and economies that depend on them.[2] Understanding how populations of marine life can be harvested sustainably is vital to the economies of nations and to the well-being of millions. We seek to gain insight into how people can use fish as a resource and produce maximum economic benefit while maintaining sustainable marine populations. Using a modified logistic growth model with a limiting equilibrium population and a threshold population, we represented a generalized fish population in order to determine sustainable fishing rates. We find that 1) increasing the fishing rate will decrease the equilibrium population and increase the threshold population; 2) for a given fish population, there exists some fishing rate that optimizes economic benefit and the growth rate of the fish population; 3) for a given population, there exists a fishing rate at which the threshold and equilibrium populations will be equal; and 4) fishing above will cause the fish population to decline to zero. We apply our findings to a threatened population of fish, the Chinook salmon that spawn in the Lower Skagit River of Puget Sound in Washington State, and issue recommendations for maintaining the security of this population.

Problem Statement

For centuries, cod fishing was a mainstay of the economy of Newfoundland in Canada. The region was settled, in fact, because of its vast supplies of this and other Atlantic fish. In 1992, however, cod levels had declined so drastically that the Canadian government placed a moratorium on cod fishing in the area. Data suggested that as much as 60% of the adult cod population had been harvested for several years in a row. The sudden drop in fish populations was detrimental to the economy of the region. As a consequence, for months in 1996, the Burin Peninsula in Newfoundland had the highest unemployment rate in Canada. Cod, which had seemed an inexhaustible resource, had not been inexhaustible after all.[3]

The situation in Newfoundland is a dramatic example of the detrimental effects of excessive exploitation of marine resources. Other fish populations throughout the world are currently in danger of similar overexploitation. According to the United States National Oceanic and Atmospheric

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[1] UN News Centre, “Overfishing: a threat to marine biodiversity.”
[2] Pollack, Susan, “No more fish stories. (economic impact of overfishing).”
[3] E – The Environmental Magazine, “A Run on the Banks: How ‘Factory Fishing’ Decimated Newfoundland Cod.”

Administration (NOAA), several populations of Chinook salmon, a Pacific species fished commercially and recreationally, are currently endangered, meaning that the population is in danger of extinction, or threatened, meaning that the population is likely to become endangered.[4]

The Chinook salmon *(Oncorhynchus tshawytscha)* is a fish of ecological and economic importance. Also known as king salmon, it is harvested commercially, prized by recreational sport fishermen, and caught as a source of subsistence by many native peoples in the Pacific Northwest region. Commercial fishing of Chinook salmon is a multimillion-dollar industry on the Pacific coast of the US, Canada, Japan, and Russia.[5]

The state fish of Alaska, the Chinook salmon is native to the Pacific Ocean, ranging as far south as the US border with Mexico and as far north as the Bering Sea.[6] Chinook salmon live most of their lives in the ocean, but return to freshwater streams to spawn, after which they die. The newly hatched salmon mature for a year in freshwater streams and pools. They then migrate to the ocean to live, on average, another two to four years before returning to the inland streams where they hatched to spawn and die.[7]

Chinook salmon are a vital part of ecosystems in the Pacific Ocean and in the streams where they spawn. In the ocean, salmon are a dominant predator, feeding on crustaceans, squid, krill, and other fish. Adult salmon are a food source for marine mammals such as orca and sea lions, as well as bears and large birds of prey.[8]

The Chinook salmon of the Puget Sound region are currently considered threatened under the Endangered Species Act.[9] We examine a specific population within Puget Sound, the Chinook salmon that spawn in the Lower Skagit River. We model the effects of fishing on this population in order to determine a fishing rate that will maximize economic benefit while maintaining the salmon population at a sustainable level.

Model Design

In our preliminary report, we have put forth a few assumptions about the behavior of the fish population. We assumed that the only factor changing the equilibrium states of the fish population was the rate of harvesting by humans. In the absence of human fishing, the equilibrium states of the fish

[4] Office of Protected Resources, “Marine/Anadromous Fish Species Under the Endangered Species Act.”
[5] Delaney, Kevin, “Chinook Salmon.”
[6] National Marine Fisheries Service, “FishWatch – U.S. Seafood Facts: Chinook Salmon.”
[7] Fairbanks Fish and Wildlife Field Office, “Cyber Fish.”
[8] Wildlife Library, “Chinook Salmon.”
[9] Office of Protected Resources, “Marine/Anadromous Fish Species Under the Endangered Species Act.”

population would not change. We also assumed that population-limiting factors such as disease, predation, or climate would not change over time and would not be influenced by human fishing, and that the reproductive behavior of the fish was not changed by human fishing. In addition, because a single salmon reproduces once in its lifetime, then dies, we assume that the proportion of fish that have reached reproductive maturity is inversely proportional to the average lifespan of the fish.

Our first task is to model a natural fish population not being fished upon by humans. Following our assumption that the fish population's growth rate is proportional to the population at that instant, we can form the differential equation

$$\frac{dy}{dt} = ry \quad , \qquad (1)$$

in which y is the fish population at the time t and r is the growth rate constant. However, if we solve the differential equation (assuming that the population at t=0 is y_0), we get

$$y = y_0 e^{rt} \quad , \qquad (2)$$

which predicts that the fish population would grow exponentially for t>0. However, we do know that there are many factors (e.g. food supply, oxygen level, predators, etc) limiting the extent to which a fish population can grow. Therefore we need a model that takes into account the combined effect of these limiting factors.

In our preliminary report, we started with logistic model. [10] The differential form of this model's basic form is

$$\frac{dy}{dt} = ry(1 - \frac{y}{K}) \quad , \qquad (3)$$

for $y > 0$, $K > 0$, where K is the carrying capacity for the environment. From here, we can see that when $y < K$, $\frac{dy}{dt}$ is positive; therefore the population grows. In particular, when $y << K$, the term y/K becomes small and the differential equation essentially becomes that of the simple exponential model

[10] Boyce (2005), pg 71

(1). Conversely, when *y>K*, $\frac{dy}{dt}$ then becomes negative. Furthermore, the equation predicts a steady state at *y=K*, meaning that when the population is at the equilibrium level, it will not change. These findings suggest that any deviation from the equilibrium population designated by *K* will result in a growth rate (or decline rate for *y>K*) that will restore the population back to *K*. Below is a graph of the solved form of equation (3) taken from Boyce and DiPrima[11]:

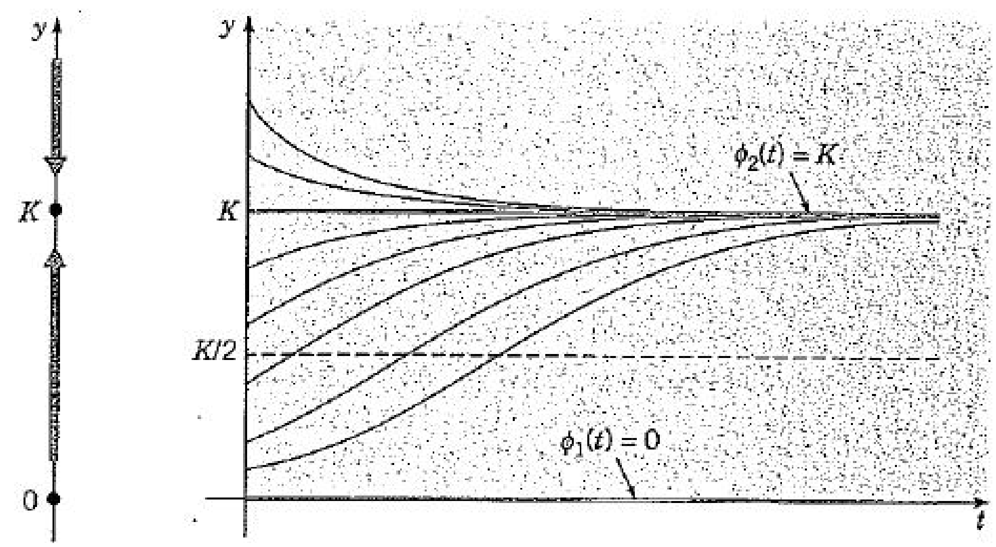

Figure 1 (from Boyce and Diprima)**:** This graph is actually a superposition of the family of solutions for equation (3). The actual "path" depends on the initial value, y_0, as well as the value of the rate constant, *r,* and equilibrium population, *K*.

The solved form for equation (3), obtained using the method of separation of variables, is

$$y = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}, \qquad (4)^{12}$$

$y_0 > 0,\ K > 0.$

As we can see from the graph, no matter where the graph is at *t*=0, the value of *y* will approach *K* as *t* increases. This is consistent with the result of taking the limit on *y* in equation (4) which yields

[11] Boyce and DiPrima (2005), pg 81
[12] Boyce and DiPrima (2005), pg 82

$$\lim_{t\to\infty} y = \frac{y_0 K}{y_0} = K \qquad (5)$$

$y_0 > 0, K > 0.$

Although the basic logistic equation models the effect of limiting factors on the fish population more accurately than the exponential growth model, it still has drawbacks. It assumes that $\frac{dy}{dt}$ is always positive when y is near zero, but we know that there has to be a minimum population below which effective reproduction cannot be sustained and the population will decline irrecoverably. To take this into account, the logistic model is modified to include such a "threshold population," T, giving us

$$\frac{dy}{dt} = -ry(1-\frac{y}{T})(1-\frac{y}{K}) \qquad (6)$$[13]

for $y > 0, 0 < T < K.$

This is the graph of the family of solutions for (3), also taken from Boyce and DiPrima: [14]

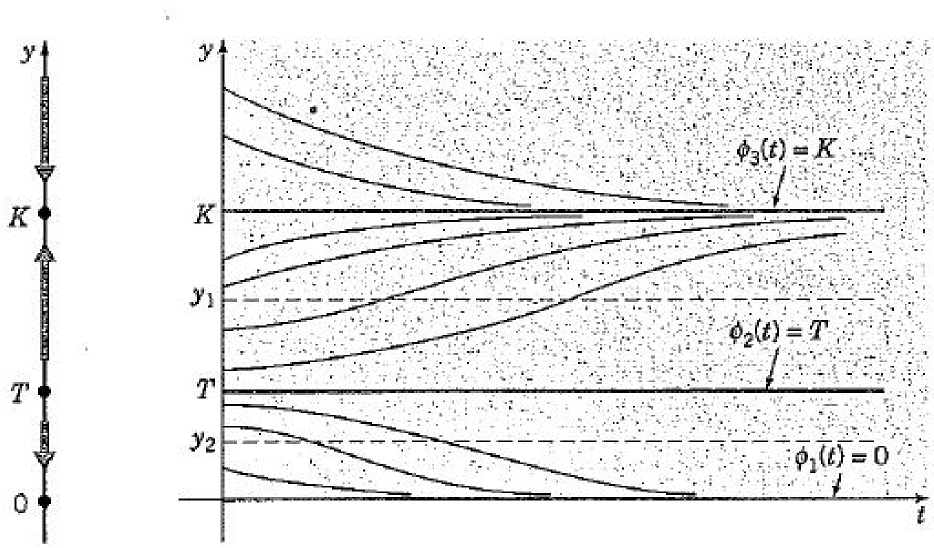

Figure 2 (from Boyce and Diprima): This is the graph of the family of solutions for equation (3). As before, *K* is the equilibrium population. *T* represents the threshold population.

[13] Boyce and DiPrima (2005), pg 86
[14] Boyce and DiPrima (2005), pg 87

It is clear from the graph that the population behaves just like the basic logistic model when *y>T*, but will decline irrecoverably once *y* is below *T*. Thus this is a more accurate representation of a natural, undisturbed fish population than either the exponential model or the basic logistic model.

Now we will bring human fishing into our model. Following the assumption that the rate of fishing is also proportional to the fish population, we designate the constant, *E,* as the fishing effort. A natural step would be to insert a term into our most accurate model, the threshold logistic equation (6), yielding

$$\frac{dy}{dt} = -ry(1-\frac{y}{T})(1-\frac{y}{K}) - Ey \quad (7)$$

for *y > 0, 0 < T < K.*

For clarity, we shall refer to *r*, *T* and *K* as the “parameters” of a given fish population from this point onwards.

Due to the tedium of solving equation (7) analytically with separation of variables, we used the following approximating equation in our preliminary report:

$$\frac{dy}{dt} = \left[r(1-\frac{y}{K}) - E \right] y \quad (8)$$

for *y > 0, K > 0,* which is essentially the incorporation of the human fishing rate *Ey* into the basic logistic equation (3). In this final report, we had, instead, modeled equation equation (7) numerically by programming an Euler iteration using the computational software Matlab.[15] We found the various steady states of *y* and *E*, and then we used Matlab’s plotting function to investigate the behavior of fish population as *y* and *E* deviate from these steady states.

To find a steady state in which human fishing combines with fish growth (represented by *ry*) and other limiting factors to result in equilibrium, we set $\frac{dy}{dt}$=0, yielding

$$0 = -ry(1-\frac{y}{T})(1-\frac{y}{K}) - Ey, \quad (9)$$

for *y > 0, 0 < T < K.*

Since we restrict our *y* to be more than 0, we can divide equation (9) throughout by *y*:

[15] See Annex A for the program code

$$0 = -r(1-\frac{y}{T})(1-\frac{y}{K}) - E, \quad (10)$$

for $y > 0$, $0 < T < K$.

Solving this quadratic equation, we get two values of y:

$$y = \frac{KT\left[r(\frac{1}{K}+\frac{1}{T}) \pm \sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}}\right]}{2r}, \quad ((11)$$

for $y > 0$, $0 < T < K$.

These, in fact, give us the new limiting and threshold populations of the fish population with the rate of human fishing E taken into account. For clarity, we separate the two solutions in equation (11) and label them as y_K and y_T, respectively:

$$y_K = \frac{KT\left[r(\frac{1}{K}+\frac{1}{T}) + \sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}}\right]}{2r}, \text{ and} \quad (11a)$$

$$y_T = \frac{KT\left[r(\frac{1}{K}+\frac{1}{T}) - \sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}}\right]}{2r}. \quad (11b)$$

To show that y_K and y_T indeed behave like K and T in equation (7), we plotted two Matlab numerical graphs of equation (7) using arbitrarily chosen parameters (each color represents a given y_0):

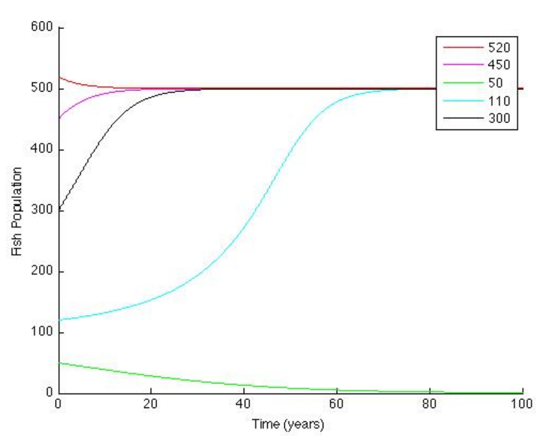

Figure 3a (r=0.05, K=500, T=100, E=0)
In this plot, we took the scenario that the fish population is not fished upon by humans. We see that for the graphs that have y_0 greater than T, the population converges to K as time increases. Also we see that for the graph with y_0 below T, the population approaches to 0 as time increases. Therefore equation (7) shows the same behavior as equation (3) did in **Figure 2**.

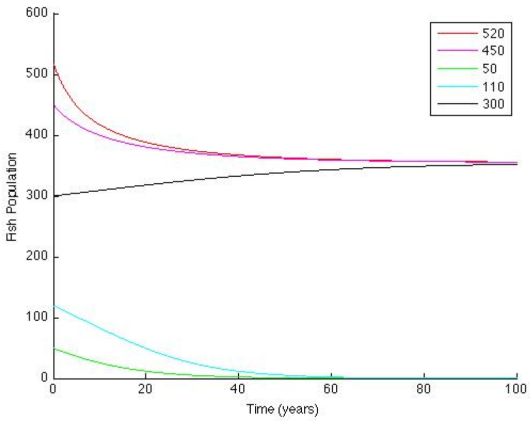

Figure 3b (r=0.05, K=500, T=100, E=0.037)
In this plot, we incorporated human fishing by setting E to be some value greater than 0. Here we see that $y_K<K$ and $y_T>T$. The graph with y_0=110, which is above the threshold in **Figure 3a**, now declines to 0. Also the calculation of y_K using equation (11a)[16] and the given parameters gives y_K =353 which is consistent with the value that graphs starting with y_0=520, 450 and 300 converge to K as time increases.

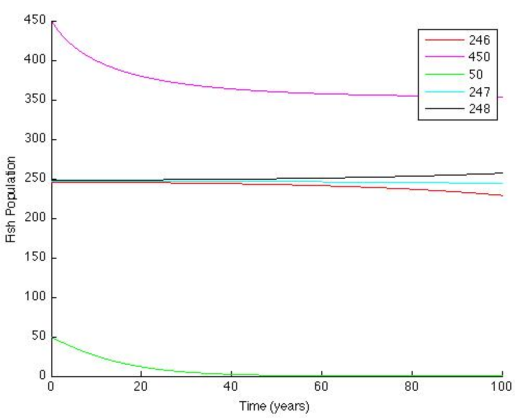

Figure 3c (r=0.05, K=500, T=100, E=0.037)
Calculation of y_T using equation (11b) and the given parameters gives y_K =247 which is consistent with fact that the solution with y_0=247 stays in equilibrium while solutions that start with y_0 slightly above or below 247 diverge from the equilibrium.

At this point, we can see that both y_K and y_T are dependent on E. multiplying y_K by E gives us Y_E, the sustainable rate of fishing (in absolute number) for that particular value of E. the equation is as follows:

$$Y_E = \frac{KTE\left[r(\frac{1}{K}+\frac{1}{T}) + \sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}} \right]}{2r}. \qquad (12)$$

To find if there is a particular value of E such that Y_E is the maximum for the given parameters, we obtained the first derivative of Y_E with respect to E:

[16] The calculation was done using the Solve[] function of Mathematica. See Annex B for details.

$$\frac{dY_E}{dt} = -\frac{E}{\sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}}} + \frac{KT\left[r(\frac{1}{K}+\frac{1}{T}) + \sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}}\right]}{2r}. \quad (13)^{17}$$

Setting $\frac{dY_E}{dt}$ to 0 and solving for *E*, we get

$$E_m = \frac{K^2r - 4KTr + rT^2 + r\sqrt{K^4 + K^3T + KT^3 + T^4}}{9KT}. \quad (14)^{18}$$

in which we denote the stationary value of *E* as E_m.

Correspondingly, we find the maximum sustainable fishing rate, in absolute numbers, is given by:

$$Y_m = E_m y_K(E=E_m). \quad (15)$$

When we examine equation (11), we see that the absolute value of the term

$$\sqrt{r^2(\frac{1}{K}+\frac{1}{T})^2 - \frac{4r(r+E)}{KT}} \quad (16)$$

gets smaller as *E* increases. By subtracting equation (11b) from (11a), we also notice that the term (16), multiplied by 2, gives the difference between y_K and y_T. From this, we deduced that when *E* is such that the term (16) becomes zero, y_K and y_T merge into one steady value y_{sm} which is stable from above but not from below. In other words, y_{sm} is a "semi-stable" equilibrium state.

Equating the term (16) to 0 and solve for the *E* (denoted as E_{sm}), we have

$$E_{sm} = \frac{r(K^2 - 2KT + T^2)}{4KT}. \quad (17)$$

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[17] The differentiation of this equation and subsequent differentiations were done using the f'[x] function of the software Mathematica. See Annex B for details.
[18] The solution equation (13) was done using the Solve function of the software Mathematica. See Annex B for details.

and the corresponding absolute fishing rate is

$$Y_{sm}=E_{sm}y_K(E=E_{sm}). \qquad (18)$$

We can take the E_{sm} to be the absolute maximum fishing rate that is still capable of maintaining a sustainable fish population in our model. In the light of this, the act of overfishing would just be the same as increasing E above E_{sm}, which has the effect of driving all solutions to the model to zero, as illustrated by the following plots from our Matlab Euler iteration program.

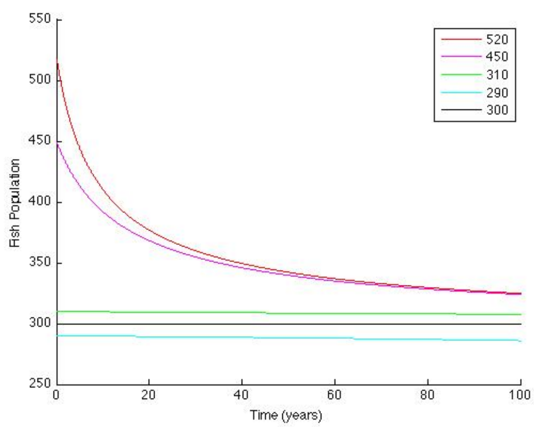

Figure 4a (r=0.05, K=500, T=100, E=0.04)
Semi-stable equilibrium: The E_m and $y(E=E_m)$ are calculated to be 0.04/year and 300 respectively. We see that for the solution with y_0=310 declines but stays above the limit of 300, whereas the solution with y_0=290 declines to zero as time increases.

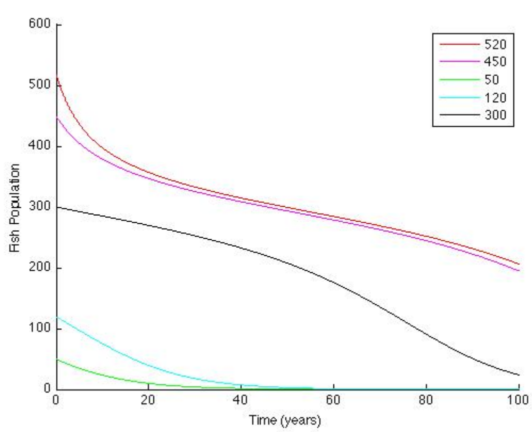

Figure 4b (r=0.05/year, K=500, T=100, E=0.045/year)
In this plot, deliberately increased E to be above E_m. Here we plainly see that all the solutions, including those with y_0 above the semi-stable equilibrium population, declines to 0 over time.

Model Implementation and Analysis

With our model in place, we shall now examine a real fish population, the Chinook salmon population that spawns in the Lower Skagit River of Puget Sound in the state of Washington. For the returning spawners of this particular population of salmon, we have found the following population data as the closest approximation of the parameters in our model:

Average number of returning spawners: 2833/year;[19]
Historical Maximum number of returning spawners: 4866/year;[20]
Historical Minimum number of returning spawners: 1043/year;[21]
Average growth rate: 1.05.[22]

Incorporating these raw data, we set our parameters as:

r=1.05;
T=1043;
K=4866,

with an initial value of y_0 = 2833.

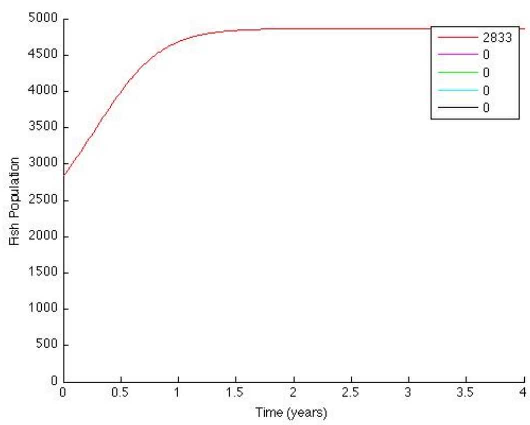

Figure 5 (r=1.05, K=4866, T=1043, E=0) Our model predicts that given the current number and parameters, the salmon population in Lower Skagit will reach its limiting value in the absence of human fishing.

By applying equations (14), (11a), (11b), (17) and (18), we get the following critical values for the salmon spawner population:

E_m=0.705;
$y_K(E=E_m)$=3448;
$y_T(E=E_m)$=2460.2;
Y_m=3448x0.705=2431;
E_{sm}=0.756;

[19] NOAA Technical Memorandum Update (2005), pg 69
[20] NOAA Technical Memorandum Update (2005), pg 69
[21] NOAA Technical Memorandum Update (2005), pg 69
[22] NOAA Technical Memorandum Update (2005), pg 82

$y_K(E=E_m)$=2954.

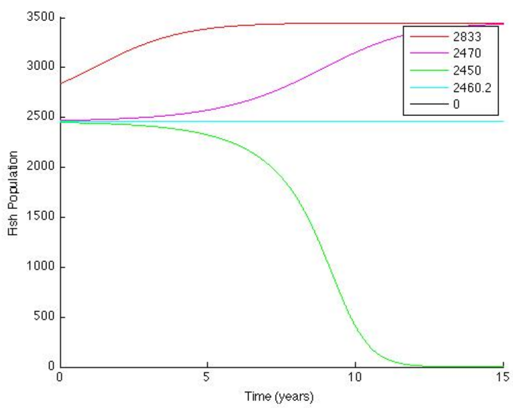

Figure 6 (*r*=1.05, *K*=4866, *T*=1043, *E*=0.705) With fishing rate maintained at 2431/year, the salmon spawner population will be maintained at 3448/year, well above the threshold at 2460.2/year.

In addition, we also found that for the salmon spawners in Lower Skakit, the earliest recorded fishing rate is 0.86, well above the semi-stable fishing rate of 0.756.[23] We incorporated this rate to our model to see the extrapolated effect on the salmon population.

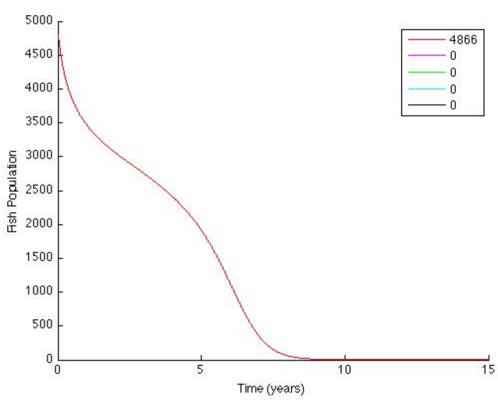

Figure 7 (*r*=1.05, *K*=4866, *T*=1043, *E*=0.86) **Historical overfishing**: here we see that had there been no measures to lower the rate of fishing, this exceedingly high overfishing would have driven the Lower Skagit salmon population (which started out at its natural limiting population of 4866) to extinction within 10 years.

Discussion

From solving our equations and inputting arbitrary data, we have reached the following conclusions:

1. If the fish population goes below the effective threshold population for a specific fishing effort, then the population will continue to decline even if fishing stops. There is also an effective

[23] NOAA Technical Memorandum Update (2005), pg 95

limiting equilibrium population for a specific fishing effort. This means that as long as a fish population is not below the effective threshold then the fish population will approach the effective limiting population over time.

2. As the fishing effort increases, the effective threshold population will increase and the effective limiting population will decrease. This result implies that as fishing effort increases, the fish population will be more susceptible to population declines, meaning that a population is more likely to go extinct.
3. If a fish population, is less than the effective threshold but still greater than the natural threshold (given by the parameter *T*), then the fish population can still recover. As long as a population of fish is not fished beyond its original threshold population then a fish population can still recover.
4. For any constant set of parameters (*r*, *K*, *T*) there exists a fishing rate such that there is a maximum sustainable harvest.
5. As the fishing effort increases, the difference between the effective threshold and limiting populations decreases until it reaches zero at a "semi-stable" equilibrium. Furthermore, for any fishing effort greater than the effort at the semi-stable equilibrium, the fish population will decline to zero over time regardless of the initial fish population. This is to say that for any population, there exists some greatest fishing effort which still allows for a sustainable population. Any effort greater than this value regardless of the initial population will drive the population to extinction.

We applied our math programs to a real set of data. Using a similar strategy as the one for arbitrary values, we found that our conclusions held up for a realistic population. In the NOAA report, we found that historically, the fishing rate was much higher than both our predicted semi-equilibrium fishing effort. Our model shows that had this fishing rate been maintained, the salmon population in Lower Skagit River would have died out within ten years. Furthermore, from these conclusions and the real data from the NOAA report, we were able to come up with a few suggestions in order to create a sustainable population:

1. A thorough study of the fish population must be conducted. The study should find accurate approximations of the parameters that are necessary for our model to calculate an optimal rate of fishing. Furthermore, an estimate of the initial fishing population must be made.
    a. If the study finds that the initial population is less than, the effective threshold population for the maximum sustainable fishing rate, then there should be no fishing conducted on the population until the population goes above the effective threshold.
    b. If the initial population is greater than the effective threshold, then start with a fishing effort no greater than the maximum sustainable rate.
2. A fish census must be performed at least yearly in order to determine current population. If one finds that the population goes below the effective threshold, then fishing should be stopped entirely before the population goes below T.
3. For short-term economic benefits, it is possible to allow a short term increase in fishing effort to be greater than the maximum fishing effort but the following measures must be taken:
    a. The short-term rate must be less than the semi-stable equilibrium effort.
    b. Fish population must be monitored more closely, maybe a census each month.
    c. The fishing rate must be returned to the maximum sustainable rate well before the population declines to the "semi-stable" equilibrium population.

Using these measures, we believe it is possible for there to be human economic benefits while still maintaining a stable sustainable fish population.

With any model, our model included, the greatest weaknesses are the assumptions made. Our model for example assumes that there are no outside factors, which impact a fish population. In reality, however, this is impossible. A fish population will be impacted by factors such as disease, cleanliness of water, availability of food, human impacts, as well as many others. None of these factors were included in our model. A future model may include some of these more damaging factors so that a better model can be created.

The model also assumes constant reproductive and fishing rates, which is not necessarily true in nature. Fish will not reproduce at a constant rate year to year due to various environmental factors. Similarly, the fishing effort will not necessarily be constant since there are many factors impacting how much fish may be caught in one period. Future models should allow for the possibility of changes in the reproductive and fishing rates.

Furthermore, this model does not include the replenishment of natural fish populations with nursery populations. Allowing for replenishment will cause the fish population to change from year to year, which would also change the maximum sustainable fishing effort. Including these values will greatly complicate matters since all the parameters will fluctuate year-to-year depending on how many fish are added to the natural population. Future models may include these values in calculating the maximum sustainable fishing effort. If there is a constant replenishing rate year to year, then a model can be designed to include the replenishing rate.

Despite these shortcomings, our model is still a strong model of a fish population. The conclusions derived from this model are still sound even if one is to consider all the other potential factors impacting a fish population.

Conclusion

Using our model, we were able to come to many important conclusions. For example, we were able to find the impact of fishing on a certain fish population. We were also able to find that there exists a fishing rate, which allows for maximum economic benefit as well as a sustainable fishing population. Furthermore, we were able to apply our data to a realistic fish population and find, at the current fishing rate, the population to be sustainable. Our models have led to many important conclusions and suggestions regarding fishing. They all lead to the overarching fact that it is extremely possible for humans to have economic benefits from fishing while still maintaining a sustainable fishing population as long as a few important measures are taken.

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11. National Oceanic and Atmospheric Administration (NOAA) Technical Memorandum NMFS-NWFSC-66 (June 2005) "Updated Status of Federally Listed ESUs of West Coast Salmon and Steelhead". Edited by Thomas P. Good, Robin S. Waples, and Pete Adams

Softwares used:

1. Wolfram Mathematica for Students, Version 7.01
2. Matlab Version 7.9.0.529 for 64 bit Mac ©The Mathworks Inc.

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Annex A: Source Code for the Matlab Euler Iteration Program

1. Original program written by Dr E. M. Strawbridge

```
clear

% Y=fish population
% t=time in days
% r=intrinsic repro rate
% T=threshold population
% K=limiting population
% E=rate of fishing

r=0.05;
T=100;
K=500;
E=0;

Tot_time=100;
dt=0.01;
t = (0:dt:Tot_time);
Y = zeros (size (t));

tot_steps=Tot_time/dt+1;
ic (1)=520;
ic (2)=450;
ic (3)=50;
ic (4)=120;
ic (5)=300;

for j=1:5
Y (1)=ic (j);

for i = 1:tot_steps-1

Y (i+1) = Y (i)+dt*(-r*Y (i)*(1-Y (i)/T)*(1-Y (i)/K)-E*Y (i));

end

figure (1)
hold on
if (j==1)
plot (t,Y,'r')
elseif (j==2)
plot (t,Y,'b')
elseif (j==3)
plot (t,Y,'g')
elseif (j==4)
plot (t,Y,'y')
```

```
else
plot (t,Y,'k')
end

end
legend ('520','450','50','120','300')
xlabel ('Time')
ylabel ('Fish')

% legend ('Fish with i.c. 700')
```

2. Modified program for model analysis using arbitrary parameters (the following is the exact code used to produce **Figure 3a**)

```
clear

% Y=fish population
% t=time in years
% r=intrinsic repro rate
% T=threshold population
% K=limiting population
% E=rate of fishing

r=0.05;
T=100;
K=500;
E=0;

Tot_time=100;
dt=0.01;
t = (0:dt:Tot_time);
Y = zeros (size (t));

tot_steps=Tot_time/dt+1;
ic (1)=520;
ic (2)=450;
ic (3)=50;
ic (4)=110;
ic (5)=300;

for j=1:5
Y (1)=ic (j);

for i = 1:tot_steps-1

Y (i+1) = Y (i)+dt*(-r*Y (i)*(1-Y (i)/T)*(1-Y (i)/K)-E*Y (i));

end
```

```
figure (1)
hold on
if (j==1)
plot (t,Y,'r')
elseif (j==2)
plot (t,Y,'m')
elseif (j==3)
plot (t,Y,'g')
elseif (j==4)
plot (t,Y,'c')
else
plot (t,Y,'k')
end

end
legend ('520','450','50','110','300')
xlabel ('Time (years)')
ylabel ('Fish Population')

% legend ('Fish with i.c. 700')
```

3. Modified program for the model implementation on Lower Skagit Chinook salmon spawner population (the following is the exact code used to produce **Figure 5**)

```
clear

% Y=fish population
% t=time in years
% r=intrinsic repro rate
% T=threshold population
% K=limiting population
% E=rate of fishing

r=1.05;
T=1043;
K=4866;
E=0;

Tot_time=4;
dt=0.01;
t = (0:dt:Tot_time);
Y = zeros (size (t));

tot_steps=Tot_time/dt+1;
ic (1)=2833;
ic (2)=0;
ic (3)=0;
ic (4)=0;
ic (5)=0;
```

```
for j=1:5
Y (1)=ic (j);

for i = 1:tot_steps-1

Y (i+1) = Y (i)+dt*(-r*Y (i)*(1-Y (i)/T)*(1-Y (i)/K)-E*Y (i));

end

figure (1)
hold on
if (j==1)
plot (t,Y,'r')
elseif (j==2)
plot (t,Y,'m')
elseif (j==3)
plot (t,Y,'g')
elseif (j==4)
plot (t,Y,'c')
else
plot (t,Y,'k')
end

end
legend ('2833','0','0','0','0')
xlabel ('Time (years)')
ylabel ('Fish Population')

% legend ('Fish with i.c. 700')
```

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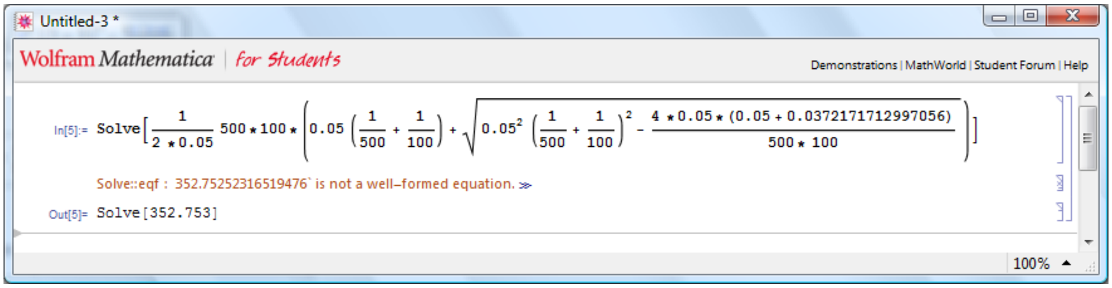
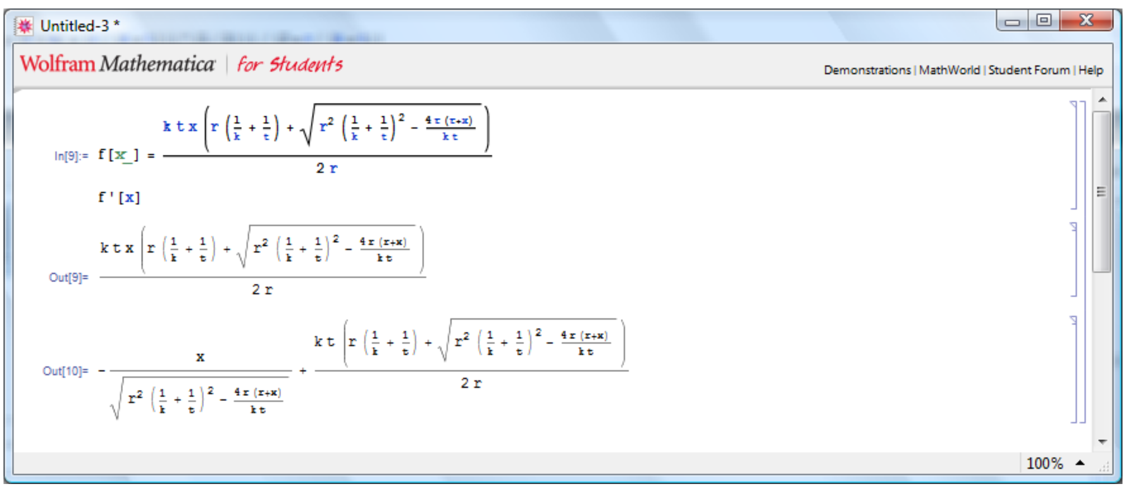
Annex B: Functions Used in Mathematica

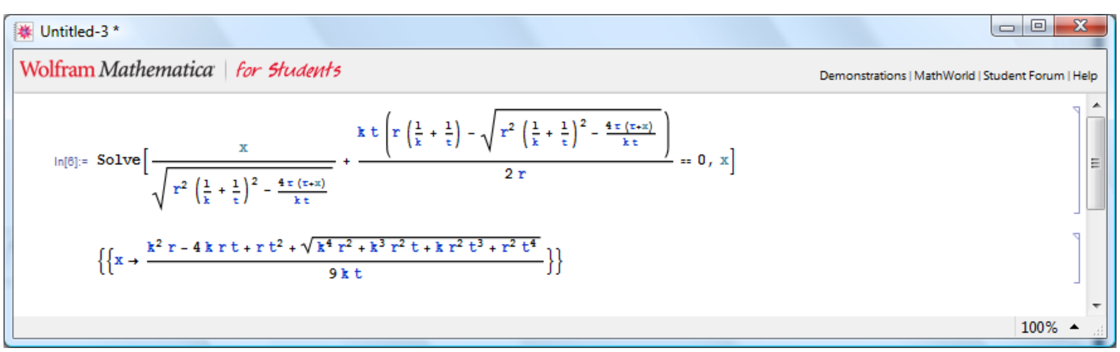
1. Solve [] function used in solving for y_K using Equation 11a in **Figure 3b**



2. f'[x] function used in obtaining equation (13) by setting equation (12) to 0 and differentiate once (E is represented by x)



3. Solve [f(x)==c, x] function used to obtain equation (14) from setting equation (13) to 0 (E is represented by x)



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Author Contributions

Bailey Steinworth

- Background literature research
- Chinook salmon population data
- Abstract writing and grammar proof-reading

Yuhui Wang

- Found analytic solutions to model equations
- Computed numerical values
- Making physical interpretations of model analysis

Xing Zhang

- Formulated model theory
- Modified Matlab iteration program
- Plotted graphs of population behavior using Matlab program

Thank you to Dr. Eva Strawbridge for writing the program used to plot graphs of the population behavior.