Homework for Week xxx **Math 231 Fall 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 4.2 2, 4, 8, 11, 15, 18, 20, 25, 26, 34, 38, 39, 43, 52, 54, 57, 59, 62.

Section 4.3 1, 3, 4, 7, 11, 12, 13, 14, 15, 16, 20, 25, 28, 42, 43, 44, 47, 50, 53, 54, 58, 64.

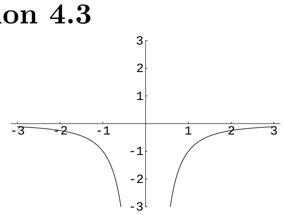
Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test.

Section 4.2

2. $f(x) = x^{\frac{1}{2}}$ is continuous on the interval $[0, \infty)$; in other words, it is continuous on $(0, \infty)$ and right but not left continuous at $x = 0$.
8. Part (a): $\lim_{x \to -1} \sqrt[6]{x} \neq \sqrt[6]{-1}$ (neither exists).
   Part (b): There exists $\epsilon > 0$ for which there is *no* $\delta > 0$ such that $0 < |x + 1| < \delta \Rightarrow |\sqrt[6]{x} - \sqrt[6]{-1}| < \epsilon$ (this is the negation of a delta-epsilon statement).
11. $(0, \infty)$
18. Any x between -0.01 and 0 will work.
20. Any $x \in (1.292, 3) \cup (3, 6.015)$ has the property that $|x^{\frac{1}{4}} - 3^{\frac{1}{4}}| < 0.25$, so the largest possible δ is $\delta = 3 - 1.292 = 1.708$.
25. Can't use continuity since $x^{-\frac{1}{6}} = \frac{1}{\sqrt[6]{x}}$ is not continuous at $x = 0$ (it isn't defined there). However we can calculate that $\lim_{x \to 0^+} x^{-\frac{1}{6}} = \lim_{x \to 0^+} \frac{1}{\sqrt[6]{x}} \to \frac{1}{0+} \to \infty$. (The left limit is undefined.)
38. $-\infty$.
39. 0.
52. $\lim_{x \to \infty} (\sqrt{x} - x) = \lim_{x \to \infty} \sqrt{x}(1 - \sqrt{x}) \to \infty(-\infty) \to -\infty$.
54. 0.
57. $f(x)$ is continuous everywhere except at $x = 1$, since $x^{\frac{2}{3}}$ is continuous on $(-\infty, 1)$ and $2x^{-1}$ is continuous on $(1, \infty)$, but $\lim_{x \to 1^-} f(x) = 1$ while $\lim_{x \to 1^+} f(x) = 2$ (so $\lim_{x \to 1} f(x)$ DNE).
59. $\lim_{h \to 0} \frac{\sqrt{3 + h} - \sqrt{3}}{h} = \lim_{h \to 0} \frac{\sqrt{3 + h} - \sqrt{3}}{h} \left(\frac{\sqrt{3 + h} + \sqrt{3}}{\sqrt{3 + h} + \sqrt{3}} \right) = \ldots \text{(work)} \ldots = \frac{1}{\sqrt{3} + \sqrt{3}} = \frac{1}{2\sqrt{3}}$.
62. Give a delta-epsilon proof showing that: For all $\epsilon > 0$, there exists $\delta > 0$ such that $0 < x < \delta \Rightarrow |\sqrt{x} - \sqrt{0}| < \epsilon$. You will end up choosing $\delta = \epsilon^2$.

Section 4.3

3. 

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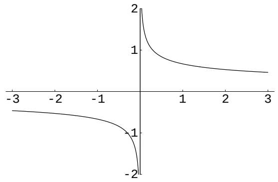
Section 4.3 (continued)

4. 

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7. The power rule does *not* apply; why not?

12. Part (a): $f'_+(c) = \lim_{h\to 0^+} \dfrac{f(c+h)-f(c)}{h}$, which is not necessarily equal to $\lim_{x\to c^+} f'(x)$. The first quantity is the limit of slopes of secant lines from $(c, f(c))$ to $(c+h, f(c+h))$ as $h \to 0^+$. The second quantity is the limit of the derivative function $f'(x)$ as $x \to c^+$.
Part (b): One example is $f(x) = x^2$, $c = 1$. The slopes of secant lines (from the right of $c = 1$) approach 2 (you can use the definition of derivative to see why), and the limit of the derivative $f'(x) = 2x$ as $c \to 1^+$ is also equal to 2.
Part (c): One example is $f(x) = \begin{cases} x^2, & x \le 1 \\ 3, & x > 1, \end{cases}$ $c = 1$. The right derivative $f'_+(1)$ does not exist. On the other hand, the limit $\lim_{x\to 1^+} f'(x)$ is equal to zero, since $f'(x) = 0$ for all $x > 1$.

14. Yes.

16. Any function of the form $f(x) = \frac{4}{3}x^3 + C$, where C is a constant, will work.

20. $\lim_{h\to 0} \dfrac{(0+h)^{\frac{2}{3}} - 0^{\frac{2}{3}}}{h} = \ldots(\text{work})\ldots = \lim_{h\to 0} \dfrac{1}{\sqrt[3]{h}}$, which approaches ∞ as $x \to 0^+$ and $-\infty$ as $x \to 0^-$. Therefore $f'(0)$ does not exist in this example.

28. $\lim_{h\to 0} \dfrac{3\sqrt{x+h} - 3\sqrt{x}}{h} = \ldots(\text{work})\ldots = \dfrac{3}{2\sqrt{x}}$.

42. $\frac{d}{dx}((3x^{\frac{1}{2}}+1)^2) = \frac{d}{dx}(9x + 6x^{\frac{1}{2}} + 1) = 9 + 3x^{-\frac{1}{2}}$.

43. Can't do this one yet; why not?

50. $f'_-(-1) = \lim_{h\to 0^-} \dfrac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^-} \dfrac{-(-1+h)-1}{h} = -1$,
$f'_+(-1) = \lim_{h\to 0^+} \dfrac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^+} \dfrac{\frac{1}{-1+h} - 1}{h} = \ldots(\text{work})\ldots =$
$\lim_{h\to 0^+} \dfrac{2-h}{h(-1+h)} \to \dfrac{2}{0^-} \to -\infty$; therefore $f'(-1)$ does not exist.

53. TYPO: The second inequality should obviously be $x \ge 1$.
$f(x)$ is continuous at $x = 1$ since the left and right limits both equal 1 (thus the limit of $f(x)$ as $x \to 1$ is equal to 1), and $f(1) = 1$. Therefore $f(x)$ at least has a chance of being differentiable. Now $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} \frac{d}{dx}(x^3) = \lim_{x\to 1^-}(3x^2) = 3$, but
$\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} \frac{d}{dx}(x) = \lim_{x\to 1^+} 1 = 1$; therefore $f(x)$ is *not* differentiable at $x = 1$.

54. $f(x)$ is not continuous, and thus not differentiable, at $x = 0$.

58. $f(x) = x^{-\frac{1}{3}}$ and $c = 8$. Since $f'(x) = -\frac{1}{3}x^{-\frac{4}{3}}$, the limit is equal to $f'(8) = -\frac{1}{3}(8)^{-\frac{4}{3}} = -\frac{1}{48}$.

64. $f(x) = \frac{3/2}{1/5}x^{\frac{1}{5}} + C = \frac{15}{2}x^{\frac{1}{5}} + C$ for some constant C. Since $f(1) = 2$ we know that $\frac{15}{2}(1)^{\frac{1}{5}} + C = 2$, and thus that $C = -\frac{11}{2}$. Therefore $f(x) = \frac{15}{2}x^{\frac{1}{5}} - \frac{11}{2}$.