Homework for Week 12 **Math 231 Fall 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 5.2 2, 6, 10, 13, 15, 18, 20, 21, 23, 24, 29, 39, 48, 53, 56, 64, 65, 68.

Section 5.3 1, 3, 8, 9, 12, 16, 24, 27, 33, 34, 42, 46, 48, 49, 53, 54, 56, 60, 61, 63.

Section 5.4 2, 6, 7, 11, 14, 16, 18, 22, 25, 28, 29, 31, 33, 34, 35, 38.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test.

Section 5.2

2. Polynomial functions never have asymptotes.
6. The degree of f is odd. Since there are 3 roots and 2 turning points, the degree is at least three (in fact, it's at least 5, because of the inflection point at $x = 1$). The leading coefficient of f must be negative.
10. Hint: Make the root a "double root."
13. Such a polynomial cannot exist.
18. Must have an even number of turning points (think about the "ends").
20. False.
21. True.
23. False.
24. True.
29. $\lim_{x\to\infty}(-2x^5 + 8x^4 - 6) = \lim_{x\to\infty} x^5(-2 + \frac{8}{x} - \frac{6}{x^5}) \to \infty(-2 + 0 - 0) \to \infty$.
39. $60x^2 - 24x^2 + 6$.
48. $f(x) = x - \frac{4}{7}x^7 + \frac{18}{7}$.
53. $f'(x) = 4(x-1)^2(x+2)$ (after synthetic division), and $f''(x) = 12(x-1)(x+1)$. f has a local minimum at $x = -2$ and inflection points at $x = \pm 1$.
56. $f'(x) = 4(x+2)(x^2+x+1)$ (after synthetic division; the quadratic factor is irreducible), and $f''(x) = 12(x+1)^2$. f has a local minimum at $x = -2$, but no inflection points.
64. Part (a): $a(t) = 0.024t$, $s(t) = 0.004t^3 + 400t$ (assuming initial position is $s_0 = 0$ at Venus).
    Part (b): $a_0 = 0$ thousands of miles per hour per hour, $v_0 = 400$ miles per hour, and $s_0 = 0$ miles.
    Part (c): Yes, since the velocity is always positive.
    Part (d): Approximately 140 hours (by using a calculator and tracing along the graph of $s(t)$, since $s(t) = 67,000$ is too hard to solve by hand). When it gets there it will be going about 635.2 thousand miles per hour.
65. If $a(t) = -32$ then (by antidifferentiating) $v(t) = -32t + C$, and $v(0) = v_0$ implies that $C = v_0$. Now (by antidifferentiating) $s(t) = -16t^2 + v_0t + K$, and $s(0) = s_0$ implies that $K = s_0$.
68. See the reading.

Section 5.3

1. $(x-c)^2 g(x)$
3. $x=-3$ is a double root, $x=-1$ is a (single) root, $x=2$ is a triple root. That second question doesn't make a whole lot of sense, don't worry about answering it.
8. One possible answer: $f(x)=(x-1)(x-2)(x-3)(x-4)(x-5)^3$.
9. One possible answer: $f(x)=(x-2)(x-3)(x^2+1)^2$.
12. One possible answer: $f(x)=(x+1)(x-1)(x-3)+10$.
16. True.
24. Your graph should have a quadruple root at $x=-2$, a root at $x=1$, and a double root at $x=3$, and should point "up" at the left end and "down" at the right end. There is a y-intercept at $(0,288)$.
27. Since $f(x)=(x^2+1)(3x-2)$ (factor by grouping) has an irreducible quadratic factor, we cannot use Theorem 1 to quickly sketch the graph of $f(x)$.
33. $f(x)=(x^2+2)^2$ (factor as you would a quadratic), so f has no roots. $f'(x)=4x(x^2+2)$, so the only critical point is $x=0$. $f''(x)=4(3x^2+2)$, which is never zero. f has a local minimum at $x=0$, but no inflection points. Use a calculator to check your graph.
34. $f(x)$ obviously has roots at $x=2$ and $x=-1$ (the root at $x=2$ is a double root). You'll have to multiply out $f(x)$ before you can differentiate it. $f'(x)=3x(x-2)$ and $f''(x)=6(x-1)$. f has a local maximum at $x=0$, a local minimum at $x=2$, and an inflection point at $x=1$.
42. From the graph we can see that $f(x)=A(x+2)(x-1)^3$ for some constant A, and that $f(0)=2$. By using $f(0)=2$ to solve for A we find that $A=-1$, and thus $f(x)=-(x+2)(x-1)^3$.
46. From the graph we can see that $f(x)=A(x+2)^2x(x-c)$, $f(-1)=7$, and $f(2)=32$. Thus (after solving a small system of equations) $f(x)=-2(x+2)^2x(x-\frac{5}{2})$.
48. From the graph we can see that $f(x)=ax^3+bx^2+cx+d$, $f(2)=0$, $f(1)=-5$, and $f(-1)=9$. That's only three datapoints, and we need to solve for four constants, so we need another piece of information; that piece of information is the fact that $f'(1)=0$ (since f has a horizontal tangent line at $x=0$). Thus (after solving a particularly nasty system of equations) $f(x)=2x^3-3x^2+4$.
49. One possible answer: Suppose $f(x)=A(x-1)^2(x-2)^2$ for some constant A. Since $f(0)=5$ we must have $A=\frac{5}{4}$.
53. Suppose $f(x)=ax^3+bx^2+cx+d$. Then $f'(x)=3ax^2+2bx+c$, $f''(x)=6ax+2b$, and $f'''(x)=6a$. With the given datapoints we must have $d=-5$, $c=-3$, $b=-1$, and $a=1$. (Check this answer by differentiating $f(x)=x^3-x^2-3x-5$ three times and checking the values of these derivatives at $x=0$.)
54. Suppose $f(x)=ax^2+bx+c$. Then (after solving the system of equations $4a+2b+c=5$, $4a+b=7$, $2a=2$) we must have $a=1$, $b=3$, and $c=-5$, so $f(x)=x^2+3x-5$ (check that this works).
56. Part (a): $f'(x)=0$ at $x=-2$, $x=0$, $x=2$, and $x=4$. Thus f' is at least degree four (for simplicity let's assume the degree *is* four). Since f' is negative at both "ends" of the graph, its leading coefficient must be negative.
    Part (b): $f'(x)=A(x+2)x(x-2)(x-4)$ where A is negative; for simplicity let's try $A=-1$ (there's no information given with which we could solve for A).
    Part (c): $f'(x)=-x^4+4x^3+4x^2-16x$, so $f(x)=-\frac{1}{5}x^5+x^4+\frac{4}{3}x^3-8x^2+C$ for some C.
    Part (d): Since $f(0)=25$, we know that $C=25$.

Section 5.3 (continued)

60. Part (a): The information in the problem implies that $s(5) = 100$, $s'(5) = -200$, $s''(5) = -46$, and that $s'(0) = 0$. We also know that $s(t) = at^3 + bt^2 + ct + d$ for some constants a, b, c, and d; use the datapoints to get system of equations so you can solve for these constants.

61. Part (c): Use the fact that $N(0) = 0$, $N(1) = 28$, $N(2) = 12$, and $N(3) = 18$ to solve for the coefficients of $N(t) = at^3 + bt^2 + ct + d$.

63. If $f(x) = ax^3 + bx^2 + cx + d$, then $f(0) = d$. Now $f'(x) = 3ax^2 + 2bx + c$, so $f'(0) = c$. Differentiating again, $f''(x) = 6ax + 2b$, so $f''(0) = 2b$, and thus $b = \frac{f''(0)}{2}$. Finally, $f'''(x) = 6a$, so $f'''(0) = 6a$, and thus $a = \frac{f'''(0)}{6}$.

Section 5.4

2. Part (a): There is some $\delta > 0$ so that $f(2) \geq f(x)$ for all $x \in (2 - \delta, 2 + \delta)$.
   Part (b): $f(2) \geq f(x)$ for all $x \in [-3, 5]$.

11. Part (a): $f'(x) = 12(x-1)^2(x+1)$ (you need to do some synthetic division to get that), so the critical points of f are $x = 1$ and $x = -1$. f has a local maximum at $x = 1$, but $x = -1$ is neither a local maximum nor a local minimum (by first or second derivative tests). The points $x = 1$ and $x = -1$ also happen to be the endpoints of our interval $[-1, 1]$. Since $f(-1) = 11$ and $f(1) = 5$, the global maximimum of f on $[-1, 1]$ occurs at $x = -1$ (with a value of 11), and the global minimum of f on $[-1, 1]$ occurs at $x = 1$ (with a value of 5).
    Part (b): No global maximum or global minimum on $(-1, 1)$.
    Part (c): The only interior local extrema is still $x = -1$ (a local maximum, with value 11). At the endpoints, $\lim_{x \to -3} f(x) = -261$ and $f(2) = -16$, so f has a global maximum at $x = -1$ (with value 11) on $(-3, 2]$, but no global minimum.

14. We are given that $a + b = 100$ and we want to minimize $a^2 + b^2$. This means we have to find the global minimum of $f(a) = a^2 + (100 - a)^2$. This minimum occurs when $a = 50$, and thus $b = 50$.

16. Let x be the length of the straight piece across the top (and bottom) and let y be the length of each of the vertical pieces. Then we know that $3y + 2x + \pi y = 120$ and we wish to maximize $A = xy + \pi(\frac{y}{2})^2$. After using the constraint to solve for x in terms of y, and simplifying, this means that $A = 60y + (-\frac{3+\pi}{2} + \frac{\pi}{4})y^2$. The only critical point of this function is $y \approx 13.1268$, and it is a local minimum of A. With this value of y we have $x \approx 19.6903$.

18. The square of the distance between $(-2, 1)$ and any point $(x, 3x + 1)$ on the graph of f depends on x, and is $D(x) = (x+2)^2 + (3x + 1 - 1)^2 = 10x^2 + 4x + 4$. The only critical point of $D(x)$ is $x = -\frac{1}{5}$, and it is a local minimum of $D(x)$. Thus $(-\frac{1}{5}, f(-\frac{1}{5})) = (-\frac{1}{5}, \frac{2}{5})$ is the point on the graph of $f(x)$ that is closest to the point $(-2, 1)$. Check that this is reasonable with a graph.

22. Hint: Maximize and minimize $v(t) = 96 - 168x^2 - 12x^3$ on $[0, 4]$.

28. TYPO: The x's in the equation for $N(c)$ should be c's!
    Part (a): Maximize and minimize $N(c)$ for $c \in [5, 55]$.
    Part (b): $R(c) = c \cdot N(C) = 0.6c^3 = 54c + 1230$.
    Part (c): Maximize and minimize $R(c)$ for $c \in [5, 55]$.

29. Maximize $V = x(4 - 2x)(6 - 2x)$ for $x \in (0, 2)$. The volume is maximized when you cut squares of side length 0.78475 from each corner.

31. Maximize $V = x^2 y$ with respect to the constraint $5x^2 + 4(5xy) + 12x^2 = 2000$. The box with the greatest possible volume has a base measuring 6.26224 inches on each side and height 10.6458 (the resulting volume is then 66.667 cubic inches). Don't forget to check that your critical point is *really* a maximum! (Use first or second derivative test.)

Section 5.4 (continued)

33. Maximize $V = x^2y$ and $S = 2x^2 + 4xy$ with constraint $y + 4x = 108$. The largest possible volume is $11,664$ cubic inches (that's not as large as it sounds!), and the largest possible surface area is $3,332.6$ square inches.
34. Maximize $V = \pi r^2 l$ and $S = 2\pi rl + 2\pi r^2$ with constraint $l + 2\pi 4 = 108$. The largest possible volume is $14,851$ cubic inches, and the largest possible surface area is 462.6 square inches.
35. Let x be the length of wire used to make the circle; then $10 - x$ is the length used to make the square. Maximize and minimize $A = \pi(\frac{x}{2\pi})^2 + (\frac{10-x}{4})^2$ for $x \in [0, 10]$. Minimum area occurs when you cut the wire so that 4.399 inches of wire are used to make the circle. The maximum area occurs when you don't cut the wire at all, and use all 10 inches to make the circle.
36. Maximize $A = xy$ with respect to the constraint $2x + 2y = P$ (remember that P is a *constant* here). You should find that A is maximized when $x = \frac{P}{4}$ and $y = \frac{P}{4}$, *i.e.* when the rectangle is really a square.