Homework for Week 3 **Math 231 Fall 2001**

This homework will not be collected; it is your responsibility to do as many problems as necessary to understand the material. We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 1.4 2, 3, 6, 12, 13, 16, 18, 20, 21, 23, 25, 27, 28, 34, 35, 36, 38, 40, 41, 45, 46, 52, 53, 54, 57, 59, 60, 61, 62.

Section 1.5 2, 7, 8, 12, 16, 20, 21, 23, 26, 31, 32, 35, 37, 38, 39, 41, 45, 55, 58, 68, 70, 71, 75, 81, 83, 87, 90, 92, 98, 99, 103, 109, 111, 112, 113.

Section 1.6 2, 4, 6, 7, 9, 10, 13, 14, 15, 18, 19, 24, 25, 26, 27, 28, 29, 36, 37, 40, 41, 46, 48, 53, 57, 58, 59, 62, 65, 66, 67, 68, 70, 71, 73.

Selected Hints and Answers

Caution: The hints and answers below are not full solutions. Most of them would not be considered complete on a quiz or test.

Section 1.4

16. This is an algebraic function (but *not* a rational function).
18. Hint: Argue that $\sqrt{x^2}$ is equal to x when $x \geq 0$ and equal to $-x$ when $x < 0$.
34. $-2 \leq x \leq 5$ and $-10 \leq y \leq 30$ is one good window. Your graph should show a horizontal asymptote at $y = 24$.
38. $f(-2) = (-2)^2 = 4$, $f(-1) = 2(-1) + 1 = -1$, $f(0) = 2(0) + 1 = 1$.
45. $|x^2 - 3x - 4| = \begin{cases} x^2 - 3x - 4, & x \leq -1 \\ -(x^2 - 3x - 4), & -1 < x < 4 \\ x^2 - 3x - 4, & x \geq 4. \end{cases}$
53. Hint: Graph $y = x^3$ and then use that graph to sketch the graph of $y = |x^3|$.
60. Your yearly pre-tax income t years after graduating is $I(t) = \begin{cases} 36,000, & 0 \leq t < 4 \\ 38,500, & 4 \leq t < 6 \\ 49,000, & t \geq 6 \end{cases}$.
61. The total amount of money you have earned t years after graduating is
$$M(t) = \begin{cases} 36,000t, & 0 \leq t < 4 \\ 144,000 + 38,500(t-4), & 4 < t \leq 6 \\ 221,000 + 49,000(t-6), & t \geq 6 \end{cases}.$$
You will have earned a total of one million dollars after approximately 21.9 years.
62. Part (a): \$2,700.00 and \$52,851.00.
Part (b): 15% and approximately 29.36%.
Part (c): $T(m) = \begin{cases} 0.15m, & 0 \leq m \leq 26,250 \\ 3,937.5 + 0.28(m - 26,250), & 26,250 < m \leq 63,550 \\ 14,381.5 + 0.31(m - 63,550), & 63,550 < m \leq 132,600 \\ 35,787 + 0.36(m - 132,600), & 132,600 < m \leq 288,350 \\ 91,857 + 0.396(m - 288,350), & m \geq 288,350 \end{cases}$.

Section 1.5

2. The domain is $[1, \infty) \cap [-4, 4] = [1, 4]$.
7. The domain is $[1, \infty) \cap [-4, 4] \cap \{x \mid x \neq 2, x \neq 5, x \neq -1, x \neq 1\} = (1, 2) \cup (2, 4]$
8. Not enough information.
20. $\{x \mid x \in \text{Domain}(g), g(x) \in \text{Domain}(f), g(x) \neq 0, \text{ and } h(x) \neq 0\}$.

Section 1.5 (continued)

21. $(-\infty, -1] \cup [1, \infty)$.

23. $(-\infty, -\frac{1}{5}) \cup (\frac{1}{5}, \infty)$.

26. Hint: Describe $(f + gh)(x)$ in terms of $f(x)$, $g(x)$, and $h(x)$.

32. $(g \circ f \circ g)(1) = g(f(g(1))) = g(f(0)) = g(2) = 1$.

38. Hint: You can work with each row in the table separately. For example, in the first row $f(0) + 2g(0) - h(0) = 4$ implies that $1 + 2(0) - h(0) = 4$, and thus $h(0) = -3$.
Answer: The missing entries in the first row are $-3, -2, 0, 0$. The missing entries in the second row are $-2, 3, 3, -6$. The missing entries in the third row are $-1, 2, 2, -7$.

39. Hint: This table is harder to complete because the rows of the table depend on each other. You can't fill in the missing entries "in order." Try to fill in the entries for the $f(x)$, $g(x)$, and $h(x)$ columns first. For example, since $f(g(2)) = 4$ you know that $g(2) = 1$. Next, since $g(f(h(1))) = 2$ you know that $f(h(1)) = 4$, and thus that $h(1) = 1$. Third, since $h(f(1)) = 3$ you know that $h(4) = 3$. Now you should be able to fill in the rest of the missing entries.
Answer: The missing entries in the first row are $1, 1, 1$. The missing entries in the second row are $1, 4, 3, 1$. The missing entries in the third row are $4, 1, 3, 1$. The missing entries in the fourth row are $3, 3, 1, 3$.

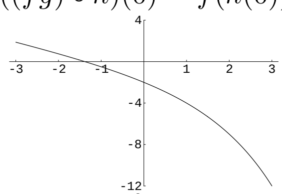
41. Part (b): $l(x) = \frac{1}{x^2}$.
Part (c): $s(t) = t$.

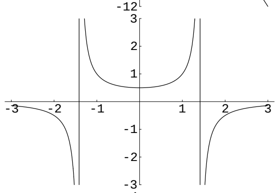
55. $(g \circ g)(x) = g(g(x)) = g(\frac{1}{x}) = \frac{1}{1/x}$ has domain $x \neq 0$. Note that $\frac{1}{1/x}$ is equal to x for all nonzero values of x, but if $x = 0$ then $\frac{1}{1/x}$ is undefined.

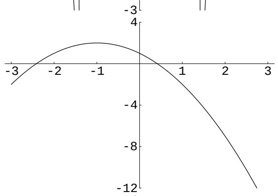
58. Hint: $((f + h) \circ g)(x) = (f + h)(g(x)) = f(g(x)) + f(h(x))$.

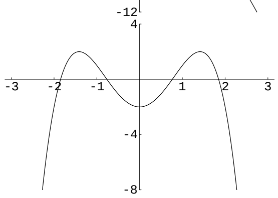
75. The row for $(f \circ h \circ h)(x)$ should be $2, 4, 3, 0, 1, 0, 3$.

81. Hint: $((fg) \circ h)(x) = (fg)(h(x)) = f(h(x)) \cdot g(h(x))$. The first entry in the row should be $((fg) \circ h)(0) = f(h(0)) \cdot g(h(0)) = f(3) \cdot g(3) = (2)(1) = 2$.

83. 

87. 

90. 

92. 

99. One possible decomposition is $g(x) = 3x + 5$ and $h(x) = x^2$. Another is $g(x) = x + 5$ and $h(x) = 3x^2$.

Section 1.5 (continued)

109. $(f+g)(x) = \begin{cases} x+1, & x \le 0 \\ x^2 - x, & 0 < x < 2 \\ x^2 + 5, & x \ge 2 \end{cases}$.

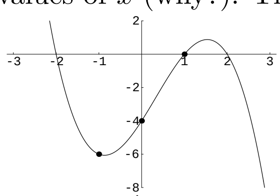
111. If $x \le 0$ then $g(f(x)) = g(2x+1) = \begin{cases} -(2x+1), & 2x+1 < 2 \\ 5, & 2x+1 \ge 2 \end{cases}$.

On the other hand, if $x > 0$ then $g(f(x)) = g(x^2) = \begin{cases} -x^2, & x^2 < 2 \\ 5, & x^2 \ge 2 \end{cases}$.

After a bit of work, this means that $g(f(x)) = \begin{cases} -(2x+1), & x \le 0 \\ -x^2, & 0 < x < \sqrt{2} \\ 5, & x \ge \sqrt{2} \end{cases}$.

Section 1.6

2. $(4, 6)$
4. $(1, 2)$
6. $(2, 1)$
7. $(-2, -5)$
14. If $g(x) = x + C$ and $h(x) = f(x)$ then $f(x) + C = g(h(x))$. The domain of this function is $\{x \mid x \in \text{Domain}(h) \text{ and } h(x) \in \text{Domain}(g)\} = \text{Domain}(f)$.
18. If $g(x) = x - 3$, $h(x) = 2x$, and $k(x) = f(x)$, then $g(h(k(x))) = 2f(x) - 3$.
25. If the first graph if $y = f(x)$ then the second graph is $y = -f(x) - 2$.
28. If f is even then we must have $f(-2) = 1$, $f(-1) = -2$, and $f(3) = 4$ (but $f(0)$ could be anything). If f is odd then we must have $f(-2) = -1$, $f(-1) = 2$, $f(0) = 0$, and $f(3) = -4$.
29. Hint: If there were such a function it would have the property that $f(x) = -f(x)$ for all values of x (why?). There is *one* function that has this property.

36.


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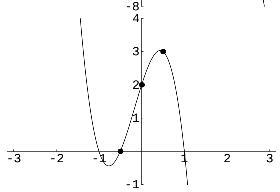

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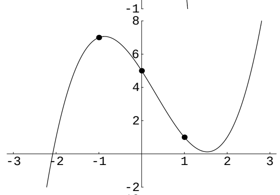
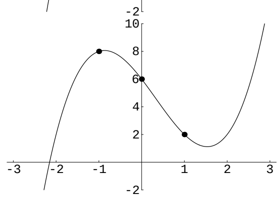


41.


Section 1.6 (continued)

58. We can only determine values of $f(2x+1)$ if the input $2x+1$ is equal to $-3, -2, -1, 0, 1, 2,$ or 3. For example, if $2x+1 = -3$ then $x = -2$, and thus when $x = -2$ we have $f(2x+1) = f(-3) = 8$ (from the first entry in the original table). The completed table is given below:

x	-2	-1.5	-1	-0.5	0	0.5	1
$f(2x+1)$	8	5	4	-2	0	3	4

65. $f(-x) = (-x)^4 + 1 = x^4 + 1 = f(x)$, so $f(x)$ is an even function.

66. $f(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$, which is neither equal to $f(x)$ nor to $-f(x)$, and thus $f(x)$ is neither even nor odd.

71. The graph of $y = f(x+3)$ is be the graph of $y = f(x)$ shifted to the left three units. We can write $f(x+3)$ as the following piecewise function:

$$f(x+3) = \begin{cases} 2(x+3)+1, & x+3 \leq 1 \\ (x+3)^2, & x+3 > 1 \end{cases} = \begin{cases} 2(x+3)+1, & x \leq -2 \\ (x+3)^2, & x > -2 \end{cases}.$$

73. Hint: Use the fact that by definition, f is an odd function if $f(-x) = -f(x)$. What does this say when $x = 0$?