**Homework for Week 8** **Math 231 Fall 2001**

*This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.*

**Section 3.5** 6, 12, 15, 16, 20, 22, 25, 29, 39, 40, 41, 50, 55, 57, 59, 63, 67, 68.

**Section 3.6** 6, 11, 15, 16, 19, 21, 25, 27, 28, 34, 38, 41, 47, 51, 57, 58, 60, 62.

**Section 3.7** 1, 4, 6, 7, 9, 14, 20, 22, 26, 29, 30, 37, 44, 47, 48, 50, 55.

## Selected Hints and Answers

*Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test.*

### Section 3.5

12. Hint: At some point in your calculation you should have $\frac{d}{dx}(3) - 2\frac{d}{dx}(x) + 5\frac{d}{dx}(x^2)$.
15. $f'(x) = -2 + 10x$.
16. $f'(x) = \lim_{h\to 0} \dfrac{(3 - 2(x+h) + 5(x+h)^2) - (3 - 2x + 5x^2)}{h} = \ldots(\text{work})\ldots = -2 + 10x$.
20. 6.
22. $-11$.
25. $f(x)$ could be the function $f(x) = x - x^2$. The function $f(x) = x - x^2 + 5$ also works.
29. $f'(x) = \ldots(\text{work})\ldots = 2(\frac{d}{dx}(1) + 3\frac{d}{dx}(x^2)) = 2(0 + 3(2x)) = 12x$.
39. $f(x) = x - 1$ (after cancellation), except that it is undefined at $x = -1$. Therefore $f'(x) = \frac{d}{dx}(x-1) = 1$, except at $x = -1$, where it is undefined (since $f$ is undefined).
40. $f(x) = x(x-3) = x^2 - 3x$, so $f'(x) = 2x - 3$.
41. $f(x) = x(x^2-3) = x^3 - 3x$ is a function we don't have a rule for differentiating (yet), because we don't have a rule for differentiating $x^3$. (We also don't have a rule for differentiating products.)
50. $f''(x) = -12$, so in particular $f''(-1) = -12$.
55. $f'(x) = -6x$, so $f'(2) = -12$.
57. $f''(x) = -8$.
59. $f''(-1) = 2$.
67. Hint: Mimic the proof of the sum rule from the reading.
68. Part (a): $$\frac{d}{dx}(ax^2 + bx + c) = \lim_{h\to 0} \frac{(a(x+h)^2 + b(x+h) + c) - (ax^2 + bx + c)}{h}$$ $$= \ldots(\text{work})\ldots = 2ax + b.$$
    Part (b):
    $$\frac{d}{dx}(ax^2 + bx + c) = \ldots(\text{work})\ldots = a\frac{d}{dx}(x^2) + b\frac{d}{dx}(x) + c = a(2x) + b(1) + 0 = 2ax + b.$$

### Section 3.6

6. Your graph should have roots at $x = -2$ and $x = 2$ and horizontal tangent lines at three places between these roots.
11. If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there exists some $c \in (a, b)$ such that the slope of the tangent line to the graph of $f$ at $c$ is equal to the slope of the line from $(a, f(a))$ to $(b, f(b))$.

**Section 3.6** (continued)

15. True by MVT, since $\frac{0-4}{2-(-2)} = -1$.
16. False; one counterexample is the function $f(x) = (x-5)^3$ (look at the graph of this function at $x = 5$).
19. One example is the graph of the function $f(x) = \begin{cases} -(x+2)^2, & x \neq -2 \\ 3, & x = 2. \end{cases}$
21. One example is an "upside-down V" with roots at $x = -2$ and $x = -2$ where the top point of the $V$ occurs at $x = -1$.
25. Hint: Draw a graph that happens to have a slight "corner" just at the place where its tangent line would have been equal to the average rate of change.
27. Hint: Use Rolle's Theorem.
28. Hint: Use the Mean Value Theorem.
34. $f'(x) = 0$ at $x \approx 0.5$, $x \approx 2$, and $x \approx 3.5$. The first point is a local minimum, the last point is a local maximum, and the middle point is neither a maximum nor a minimum.
38. $x = -0.65$.
41. From the graph, $f$ appears to be continuous on $[0, 4]$ and differentiable on $(0, 4)$, and moreover $f(0) = f(4) = 0$, so Rolle's Theorem applies. Therefore there is some $c \in (-4, 4)$ such that $f'(c) = 0$. In this example there are three such values of $c$, namely $c \approx 0.5$, $c \approx 2$, and $c \approx 3.5$.
47. Show that $f$ satisfies the hypotheses of Rolle's Theorem (see above). Then you know that there is some $c \in (-1, 4)$ such that $f'(c) = 0$. By solving $f'(x) = 0$ we can see that there is exactly one such value, $c = \frac{3}{2}$.
51. Show that $f$ satisfies the necessary hypotheses. There are two values $x = c$ that satisfy the conclusion of the Mean Value Theorem, at $c \approx -2.8$ and $c \approx -0.9$.
57. Show that $f$ satisfies the necessary hypotheses. Then by MVT there is some $c \in (-3, 2)$ such that $f'(c) = 1$ (the average rate of change of $f$ on $[-3, 2]$). Solving $f'(x) = 1$ we see that there is exactly one such value, namely $x = -\frac{1}{2}$.
58. Hint: Is $C(h)$ differentiable? Is $C'(4) = 0$?
60. Part (a): Let $s(t)$ be Alina's distance from the grocery store (in miles) at time $t$. Then $s(0) = 20$ and $s(0.5) = 0$; since $s(t)$ is continuous and differentiable (why?), the MVT applies, and tells you that there is some time $c \in (0, 0.5)$ where $s'(c) = \frac{0-20}{0.5-0} = -40$ miles per hour (the negative sign just means she is travelling towards the store).
62. See the reading.

**Section 3.7**

1. True.
4. False.
6. False.
7. False.
9. Hint: Think about the Intermediate Value Theorem.
14. Your graph of $f$ should be increasing on $(-\infty, 1)$, $(3, 8)$, and $(8, \infty)$ and decreasing on $(1, 3)$. It should also have horizontal tangent lines at $x = 1$ (a local maximum) and $x = 8$ (not an extrema), and be non-differentiable at $x = 3$ (a local minimum). Your graph of $f'$ should be positive on $(-\infty, 1)$, $(3, 8)$, and $(8, \infty)$ and negative on $(1, 3)$. It should also have roots at $x = 1$ and $x = 8$ and be undefined at $x = 3$.
20. In order: Yes. Yes. No.
26. Your graph of $f'$ should have roots at $x = 0$ and $x = 2$, and should be positive on $(0, \infty)$ and negative on $(-\infty, 0)$. In particular this means that your graph of $f'$ should "bounce" off the $x$-axis at $x = 2$.

**Section 3.7** (continued)

29. Your graph of $f$ should have horizontal tangent lines at $x = 1$ at $x = 3$, and should be decreasing on $(-\infty, 1)$ and $(1, 3)$ and increasing on $(3, \infty)$. The point $x = 1$ is an inflection point (but not a local extrema) of $f$, while the point $x = 3$ is a local minimum of $f$.
37. Increasing on $(-\infty, -\frac{1}{3})$ and $(1, \infty)$; decreasing on $(-\frac{1}{3}, 1)$.
44. $f$ has a critical point at $x = 2$ but does *not* have a local extrema there; thus $f$ has *no* local extrema.
47. Warning: Notice that the graph given is the graph of *velocity*, not position.
    Part (a): On the time interval $(0, 2.5)$.
    Part (b): At time $t = 2.5$.
    Part (c): On the time intervals $(0, 1)$ and $(3.3, 4)$.
    Part (d): Time $t = 3.8$ is such a time; Bubbles is moving towards the left side of the tunnel and is slowing down while doing so.
48. TYPO: Should say $f$ is ***decreasing*** on $I$. Mimic the proof of part (a) of Theorem 1.
55. Given that $f(x) = mx + b$ where $m \neq 0$ (so $f(x)$ is nonconstant), show that $f'(x)$ is either always positive or always negative. Since $f'(x) = m$ is a (nonzero) constant, this is clearly the case.