Homework for Week 9 **Math 231 Fall 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 3.8 5, 6, 7, 8, 10, 13, 14, 17, 19, 20, 24, 32, 39, 44, 46, 49, 56, 63, 67, 70, 71, 74, 75.

Section 4.1 3, 7, 12, 13, 15, 16, 20, 23, 35, 40, 49, 52, 57, 59, 63, 70, 74, 78, 85, 87, 89, 90, 94, 96, 98.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test.

Section 3.8

5. Hint: If f'' is zero on I, then f' is constant on I; what kind of function f has a constant derivative?
6. False.
7. True.
8. True.
10. True.
13. True.
14. Your graph should be positive, decreasing, and concave up everywhere (like the third arc in Figure 9).
17. Your graph should be increasing everywhere, concave up on $(-\infty, -2)$, and concave down on $(2, \infty)$ (and thus it should have an inflection point at $x = 2$ of the type shown in the third graph in Figure 6).
24. Part (c): Show that $f''(x) = 12x^2$ does not change sign at $x = 0$ (for example, look at $f''(-1)$ and $f''(1)$).
32. Concave up on $(-\infty, 0) \cup (1, \infty)$, concave down on $(0, 1)$.
39. $f''(x) = 0$ for $x = \frac{6+\sqrt{12}}{6} \approx 1.57735$ and $x = \frac{6-\sqrt{12}}{6} \approx 0.42265$ (use quadratic formula to solve $4(3x^2 - 6x + 2) = 0$). $f''(x)$ does change sign at each of these points (for example, $f''[0] = 8$, $f''[1] = -4$, and $f''[2] = 8$), so they are both inflection points.
44. $f'(x) = 0$ at $x = 0$ and at $x = \pm\frac{1}{\sqrt{3}}$. Since $f''(0) = -4$, $x = 0$ is a local maximum. The other two are local minimums (look at $f''(x)$ at those points).
46. f has a local maximum at $x = -2$, a local minimum at $x = 1$, and an inflection point at $x = -\frac{1}{2}$.
56. Hint: Among other things, your graph should have a local minimum at $x = -2$ and inflection points at $x = -1$ and $x = 0$.
63. Did you show your work clearly for each of the five parts of Algorithm 2?
67. Your graph of $f'(x)$ should have roots at $x = -3$ and at $x = 0$, and should be positive on $(-\infty, -3)$ and negative on $(-3, \infty)$. Your graph of $f''(x)$ should have roots at $x \approx -2$ and $x = 0$, and should be negative on $(-\infty, -2) \cup (0, \infty)$ and positive on $(-2, 0)$.

Section 3.8 (continued)

70. Your graph of $f(x)$ should have a horizontal tangent line (in fact, a local minimum) at $x = 1$, and should be decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$. The fact that the graph of $f'(x)$ has a horizontal asymptote at $y = -1$ as $x \to -\infty$ means that your graph of f should have a slope approaching -1 as $x \to -\infty$; in other words, your graph of f should be very much like a line with slope -1 as $x \to -\infty$. Your graph of $f''(x)$ should have no roots, should be always positive, and should have a horizontal asymptote at $y = 0$ as $x \to -\infty$ (since the slope of $f'(x)$ approaches zero as $x \to -\infty$).

72. Your graph of $f'(x)$ should be decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$, with a local minimum at $x = 2$. Your graph of $f(x)$ should be concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$, with an inflection point at $x = 2$.

74. Part (a): $[0, 15]$; she is north of the corner, walking north and slowing down.
Part (b): $[60, 80]$; she is south of the corner, walking south and speeding up.
Part (c): $[15, 40]$; she is walking south and speeding up (first north and then south of the corner).
Part (d): At $t = 40$; she is south of the corner, walking south; she just finished speeding up (in the southward direcction) and is now just starting to slow down (but still walking in the southward direction).

75. See the proof of part (a) of Theorem 1.

Section 4.1

3. Not a power function.

7. $f(x) = x^{-\frac{3}{2}}$.

12. Part (b): If x^0 were zero then $\frac{x^2}{x^2} = x^{2-2} = x^0$ would equal zero (instead of what we want it to equal: one).
Part (c): If x^0 were zero then we'd have $x^2 = x^{2+0} = x^2x^0 = x^2(0) = 0$, and clearly it isn't true that $x^2 = 0$ for all x!

13. Part (a): $x^0 = 1$ for *any* small nonzero value of x, so $\lim\limits_{x\to 0} x^0 = \lim\limits_{x\to 0} 1 = 1$.
Part (b): $0^k = 0$ for *any* small nonzero value of k, so $\lim\limits_{k\to 0} 0^k = \lim\limits_{k\to 0} 0 = 0$.

20. $\frac{27}{8}$.

23. $\frac{4}{9}$.

35. $(-1, \infty)$.

40. $(-\infty, -1) \cup (1, \infty)$.

49. x^{-2}.

52. $\frac{1}{16}x^2$.

57. One counterexample is $x = -2$.

59. One counterexample is $x = 1$, $y = 1$.

63. $[2, \infty)$.

70. $\dfrac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$.

74. $a = b^2$; there are extraneous solutions, for example $a = 4$, $b = -2$ is a solution to $a = b^2$ but not to the original equation.

78. $1 = a^9b$; there could be extraneous solutions with $a = 0$, but there are no solutions to $1 = a^9b$ with $a = 0$... therefore there are no extraneous solutions.

85. The graph of $f(x)$ is the graph of $y = x^{\frac{2}{3}}$ shifted left 3 units and down 2 units (so the cusp of $f(x)$ is at $(-3, 2)$).

87. The graph of $f(x)$ is the graph of $y = \sqrt{x}$ reflected over the y-axis. Note that the domain of $f(x) = \sqrt{-x}$ is $(-\infty, 0)$, since $-x$ is positive if and only if x is negative.

Section 4.1 (continued)

89. The difference of two power functions is not necessarily a power function. One counterexample is $f(x) = x^2$, $g(x) = x^3$.

90. The quotient of two power functions is a power function (at least for $x \neq 0$). If $f(x) = Ax^k$ and $g(x) = Bx^l$ then $\frac{f(x)}{g(x)} = \frac{Ax^k}{Bx^l} = \frac{A}{B}x^{k-l}$ (when $x \neq 0$).

94. See the first proof on page 395.

96. See the second proof on page 395.

98. $\left(\frac{x}{y}\right)^a = \left(x \cdot \frac{1}{y}\right)^a = x^a \left(\frac{1}{y}\right)^a = x^a (y^{-1})^a = x^a y^{-a} = x^a \frac{1}{y^a} = \frac{x^a}{y^a}$.
The justifications for the six equalities above are (in order): algebra, Rule (b) of Theorem 1, Definition 2, Rule (c) of Theorem 1, Definition 2, and algebra.