TEST I

Math 231
September 27, 2001

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 11 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (*16 points*) Short answer questions. Do not show work.

(**a**) Define what it means for a real number to be a *rational number* without referring to decimal notation.

(**b**) Is $(x, y) = (1, 4)$ a solution to the equation $y = 2 - x + 3x^5$? Why or why not?

(**c**) Express the sentence "The distance between a and -3 is greater than or equal to two" as an inequality involving an absolute value.

(**d**) If $g(x) = x^2$ and $f(g(x)) = \dfrac{1}{x^2 + 1}$, what is $f(x)$?

(**e**) Fill in the blank: If _____ is on the graph of $y = f(x)$, then $(4, 2)$ is on the graph of $y = f(x - 3)$.

(**f**) Fill in the blank: If $(2, 3)$ is on the graph of f, then _____ is on the graph of f^{-1}.

(**g**) If $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to +\infty} f(x) = 3$, and $\lim_{x \to 1^+} f(x) = \infty$, what can you say about any horizontal asymptotes of $f(x)$ (i.e., are there any, and if so, where)?

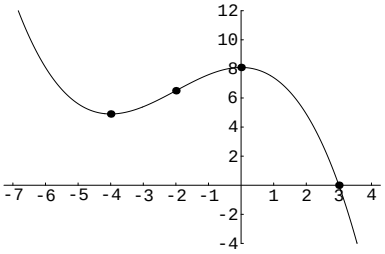
(**h**) Write the solution set of $0 < |x - 2| < 0.1$ in interval notation. (You should be able to do this without having to split the absolute value into cases.)

2. (*6 points*) Sketch a graph of the function $f(x) = \begin{cases} x^2, & x < -1 \\ 2x + 1, & x \geq -1. \end{cases}$

3. (*8 points*) Express the solution set of the inequality $|4 - 2x| > 6$ in interval notation.

4. (*8 points*) Express the domain of the function $f(x) = \dfrac{\sqrt{x}}{3x - 5}$ in interval notation.

5. (*6 points*) Given the graph of f below, fill in the blanks with the appropriate interval or intervals.



f is positive on _____ .

f is decreasing on _____ .

f is concave up on _____ .

6. (*10 points*) Use a graph of $y = 2 - x^2$ and your calculator to find the largest value of δ for which the following implication is true:

$$0 < |x - 3| < \delta \;\Rightarrow\; |(2 - x^2) + 7| < 0.01.$$

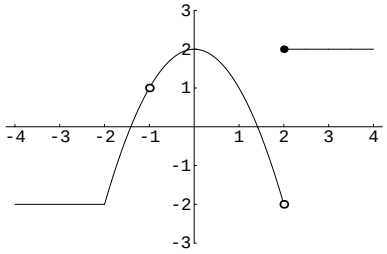
Show your work (including a labeled graph), and put your final answer in the blank provided. Make your approximation of δ accurate to three decimal places.

$\delta =$ _____ .

7. (*8 points*) Fill in the missing entries in the table below.

x	1	2	3	4
$f(x)$	2	4	4	1
$g(x)$	3	2	1	4
$(g \circ f)(x)$				

8. (*10 points*) Given the graph of $f(x)$ below, approximate each of the following limits.



$\lim_{x \to -1} f(x) =$ _____

$\lim_{x \to -\infty} f(x) =$ _____

$\lim_{x \to 2^-} f(x) =$ _____

$\lim_{x \to 2^+} f(x) =$ _____

$\lim_{x \to 2} f(x) =$ _____

9. (*10 points*) Find the inverse of the function $f(x) = \dfrac{x-1}{x+1}$.

10. (*6 points*) Write the contrapositive of the statement:

"$2 + 2 = 5$, then today is Tuesday or cows aren't reptiles."

11. (*12 points*) Label the following statements as true or false.

(**a**) **T** **F** If a is a real number, then the quantity $-a$ is negative.

(**b**) **T** **F** For all values of x, $\dfrac{(x+1)(x+2)}{x+1} = x + 2$.

(**c**) **T** **F** The set $\{x \mid x > 4 \text{ and } x < 6\}$ is equal to the set $(4, \infty) \cup (-\infty, 6)$.

(**d**) **T** **F** Define f : {People in the U.S.} $\to$ {U.S. States} by the following rule: assign every person to Kansas. This is a function.

(**e**) **T** **F** Define f : {People in the world} $\to$ {Countries} by the rule that assigns every person to the country in which they were born. This is a one-to-one function.

(**f**) **T** **F** If $\lim\limits_{x \to 1^-} f(x) = 5$ and $\lim\limits_{x \to 1^+} f(x) = 5$, then $f(1) = 5$.

Survey Question:

Were you prepared for the types of questions that were on this test? How do you think you did?

SCRAP WORK