TEST II

Math 231
October 30, 2001

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 5 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (*20 points*) Fill in the blanks or circle true (T) or false (F) for each statement below. You do not need to show any work for this problem.

a. (*1 pt*) **T F** The limit of a function $f(x)$ as $x \to c$ is equal to $f(c)$.

b. (*1 pt*) **T F** If $f'(2) = 0$ then f has a local extrema at $x = 2$.

c. (*1 pt*) **T F** If $g(x) \to 0$ as $x \to c$, then $\lim_{x \to c} \frac{f(x)}{g(x)}$ does not exist.

d. (*1 pt*) **T F** If $f'(x)$ is continuous on $(1, 8)$ and $f'(3)$ is negative, then $f'(x)$ is negative on the entire interval $(1, 8)$.

e. (*1 pt*) **T F** If $f(x)$ is continuous on the interval $(2, 4)$ then f must have a maximum value somewhere in the interval $(2, 4)$.

f. (*2 pts*) By definition, $f(x)$ is *continuous* at $x = c$ if __________________________ .

g. (*2 pts*) By definition, $f(x)$ has a *removable discontinuity* at $x = c$ if:

____ .

h. (*2 pts*) By definition, $f(x)$ is *differentiable* at $x = c$ if __________________________ .

i. (*3 pts*) State Rolle's Theorem:

If ____ ,

then ____ .

j. (*6 pts*) If $f(2) = 3$, $f'(2) = -2$, $g(2) = 5$, and $g'(2) = 2$, then:

$$\lim_{x \to 2} \frac{f(x)}{x - g(x)} = \_\_\_\_\_\_\_\_\_\_\_\_ ,$$

$$\left.\frac{d}{dx}\right|_{x=2} \left(\frac{f(x) - 3g(x)}{2} \right) = \_\_\_\_\_\_\_\_\_\_\_\_ .$$

The equation of the line tangent to the graph of $f(x)$ at $x = 2$ is ________________ .

2. (*20 points*) Calculate each of the following limits.

a. (*4 pts*) $\lim_{x \to 2} \frac{4 - 2x}{x - 2}$

b. (*4 pts*) $\lim_{x \to 1^-} \frac{1}{x^2 - 1}$

c. (*4 pts*) $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} x^2, & x < 3 \\ 6x + 1, & x \geq 3 \end{cases}$

d. (*4 pts*) Suppose $f(x)$ is the function given in part **(c)** above. Is $f(x)$ continuous at $x = 3$? Why or why not? If not, what kind of discontinuity occurs?

e. (*4 pts*) Suppose $f(x)$ is the function given in part **(c)** above. Is $f(x)$ differentiable at $x = 3$? Why or why not?

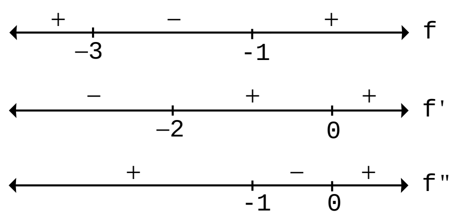
3. (*20 points*) In this problem you may ***not*** use any differentiation rules. All work must be done using the ***definition*** of derivative.

a. (*10 pts*) Use the ***definition*** of derivative to show that the derivative of $f(x) = 4 - x^2$ is $f'(x) = -2x$.

b. (*10 pts*) Prove the "sum rule" for differentiation using the definition of derivative. In other words, use the ***definition*** of derivative to prove that for any functions $f(x)$ and $g(x)$,

$$(f+g)'(x) = f'(x) + g'(x).$$

4. (*20 points*) Suppose f is a function described by the number lines below. (Note: The points marked on the number lines are the values where the functions f, f', and f'', respectively, are zero.)



a. (*16 pts*) Use the number lines above to fill in the blanks:

f is negative and increasing for $x \in$ ______________________________ .

f is positive and concave up for $x \in$ ______________________________ .

f' is decreasing for $x \in$ ______________________________ .

f has a local maximum at: ______________________________ .

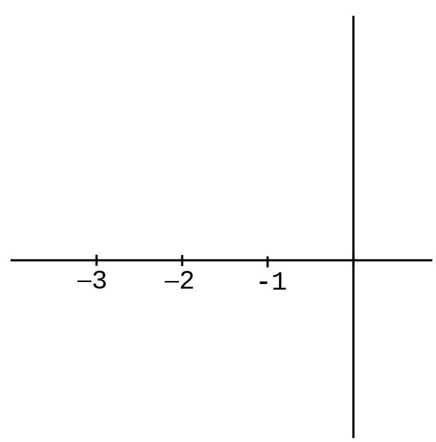
f' has a local minimum at: ______________________________ .

The critical points of f are: ______________________________ .

The inflection points of f are: ______________________________ .

The second derivative test gives no information at the critical point: ______________________________ .

b. (*4 pts*) Sketch a possible graph for f that reflects all of the information given in the number lines.



5. (*20 points*) If you drop a rotten orange from your dorm window (144 feet above the ground), its height $s(t)$ in feet above the ground t seconds after being dropped is given by the function $s(t) = -16t^2 + 144$. (You ***may*** use differentiation rules for this problem.)

a. (*4 pts*) How long does it take for the orange to hit the ground?

b. (*4 pts*) Find the average velocity of the orange during the time that it is in the air.

c. (*4 pts*) Find the instantaneous velocity of the orange at the time it hits the ground.

d. (*4 pts*) The function $s(t)$ is continuous and differentiable everywhere. What does the Intermediate Value Theorem say about the height of the orange during the time it is in the air?

e. (*4 pts*) The function $s(t)$ is continuous and differentiable everywhere. What does the Mean Value Theorem say about the velocity of the orange?

Survey Question:

What was the hardest question on this test? How do you think you did?

SCRAP WORK