

# TEST III

Math 231  
November 27, 2001

Name: \_\_\_\_\_  
By writing my name I swear by the honor code.

**Read all of the following information before starting the exam:**

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

**1.** (10 points) Let  $f(x) = 4x^{\frac{2}{5}}$ .

(a) Is  $f$  is even, odd, or neither, and why?

(b) Find the inverse,  $f^{-1}(x)$ , of  $f(x)$  on the interval  $[0, \infty)$  and write it in the form  $Cx^r$ .

**2.** (10 points) Let  $f(x) = \frac{x^{-3} - x^{-2}}{x^{-1} - 1}$ .

(a) Write  $f(x)$  in the form  $Cx^r$  (for  $x \neq 1$ ).

(b) Evaluate  $\lim_{x \rightarrow \infty} f(x)$ .

**3.** (10 points) Let  $f(x) = \frac{(4x^2)^{\frac{3}{2}}}{5x^{\frac{1}{3}}}$ .

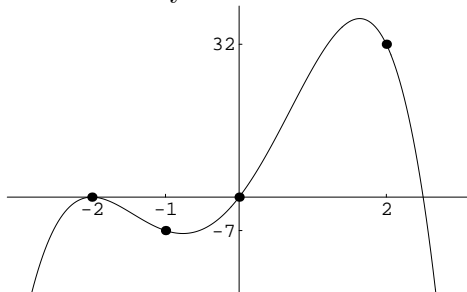
(a) Write  $f(x)$  in the form  $Cx^r$  (for  $x \neq 0$ ).

(b) Calculate  $f'(x)$ .

4. (10 points) Write the polynomial  $f(x) = 3x^3 + x - 4$  in the form  $f(x) = Ag_1(x)g_2(x)\dots g_r(x)$ , where  $A$  is a real number and each  $g_i(x)$  is either a linear factor of the form  $x - d$  or an irreducible factor of the form  $x^2 + bx + c$ .

5. (10 points) Find a cubic polynomial function  $f(x)$  such that  $f(0) = -4$ ,  $f'(0) = -2$ ,  $f''(0) = 4$ , and  $f'''(0) = 6$ .

6. (10 points) Find a polynomial function that could have the graph below. Show all of your work carefully.



**7.** (10 points) Let  $f(x) = (x^2 - 1)^{-\frac{1}{4}}$ .

(a) Find the domain of the function  $f(x)$ . Write your answer in interval notation.

(b) Find the domain of the reciprocal,  $\frac{1}{f(x)}$ , of the function  $f(x)$ . Write your answer in interval notation.

**8.** (10 points) Farmer Brown wants to build a rectangular pig pen along the edge of a river, so that she only needs to build three sides of the rectangle. If Farmer Brown has 540 feet of fence, find the dimensions (i.e., length and width) of the pig pen of maximum area. Be sure to explain why your answer yields the maximum area, and be sure to give your answer with units.

**9.** (10 points) Find all roots, holes, and asymptotes of  $f(x) = \frac{(x-1)(x-2)}{(x-1)^2(3x+2)}$ .

**10.** (10 points) Let  $f(x) = \frac{x^4 - 3x^2 + 2}{x^2 + 1}$ .

**a.** (7 pts) Use polynomial long division to write the improper rational function  $f(x)$  as the sum of a polynomial and a proper rational function.

**b.** (3 pts) What is the equation of the curve asymptote of  $f(x)$ ?

**Survey Question:**

How do you think you did?

---

**SCRAP WORK**