TEST I

Math 231
September 25, 2003

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*12 pts*) Determine whether each of the following statements is true (T) or false (F).

(a) **T** **F** $f(x) = x^2 + 4x + 6$ is an even function.

(b) **T** **F** If $x < -2$, then $|x| = -x$.

(c) **T** **F** For all x, there is some y with $x = y^2$.

(d) **T** **F** For all y, there is some x with $x = y^2$.

(e) **T** **F** If the range of $f(x)$ is $[-2, 3]$, then the range of $2f(x)$ is $[-4, 6]$.

(f) **T** **F** The graph of $f(3x)$ is three times as wide as the graph of $f(x)$.

2. (*15 pts*) Give short answers to each question below.

(a) State the formal definition of a *rational number*, without referring to decimal notation.

(b) State the formal definition of a *one-to-one* function.

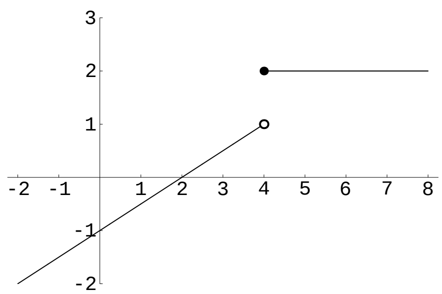
(c) Express the set below in the simplest possible interval notation.

$$(x \geq 2 \text{ and } x < 5) \text{ or } (x < 0 \text{ and } x \neq -1)$$

(d) Write down the cases involved in solving the inequality $2x^2 + 5x - 3 < 0$.
(Don't solve the inequality, just write down the cases.)

(e) Write down the cases involved in solving the inequality $|3x + 1| > 2$.
(Don't solve the inequality, just write down the cases.)

3. (*10 pts*) Write down a formula for the function $f(x)$ shown in the graph below. (Note that $f(x)$ is a piecewise function.)



4. (*18 pts*) Fill in the blanks with numbers.

(a) If $(9, 1)$ is on the graph of f, then (____ , ____) is on the graph of $f(x+3)$.

(b) If (____ , ____) is on the graph of f, then $(2, 8)$ is on the graph of $2f(x)$.

(c) If $(2, 3)$ is on the graph of an <u>odd</u> function f, then (____ , ____) is also on the graph of f.

(d) If $f^{-1}(2) = 8$, then $f(8) =$ _______ and $f(f^{-1}(2)) =$ _______ .

(e) If $f(x) = 3x + 1$, then $f(f(2)) =$ _______ and $f^{-1}(-2) =$ _______ .

(f) If $f(2) = 4$, $f(3) = 6$, $g(2) = 3$, and $g(4) = 5$, then $(f \circ g)(2) =$ _______ .

(g) If $(f + 2g)(3) = 9$ and $f(3) = 5$, then $g(3) =$ _______ .

(h) If $f(x) = \begin{cases} 1 - 3x, & \text{if } x < -1 \\ x^2, & \text{if } x \geq -1, \end{cases}$ then $f(2) =$ _______ and $f(-2) =$ _______ .

(i) If f is a <u>linear</u> function with $f(-1) = 3$ and $f(2) = 0$, then $f(3) =$ _______ .

5. (*10 pts*) A disgruntled pet store owner abandoned an unknown number of hamsters on a small island in 1996. Since then it has been determined that the average rate of change of the hamster population was 4 hamsters per year, and that the hamster population was a linear function of time. In the year 2001, when the abandoned hamsters were discovered, there were 376 hamsters on the island. How many combats did the pet store owner originally leave on the island? (Show your work so I can tell how you arrived at your answer!)

6. (*11 pts*) Consider the statement "If $x > 2$, then $x \geq 3$."

(a) Is the statement true or false?

(b) If the statement is true, explain why. If the statement if false, provide a counterexample.

(c) Write down the contrapositive of the original statement.

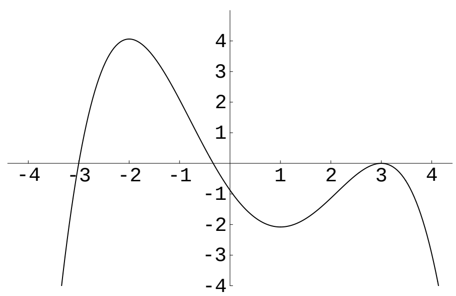
(d) Write down the converse of the original statement.

(e) Write down the negation of the original statement.

(f) Write anything you like in this box for one point: $\boxed{}$

7. (*10 pts*) Use induction to prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all positive integers n. Remember, you're writing a proof, so make sure that your argument is clear.

8. (*14 pts*) Use the graph of $f(x)$ below to fill in the blanks.



(**a**) ________________ List the roots of f.

(**b**) ________________ List the locations of any local maxima of f.

(**c**) ________________ List the locations of any global maxima of f.

(**d**) ________________ List the locations of any inflection points of f.

(**e**) ________________ List the interval(s) where f is negative.

(**f**) ________________ List the interval(s) where f is increasing.

(**g**) ________________ List the interval(s) where f is concave up.

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK