TEST I

Math 231
February 11, 2003

Name: ______________________________
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has seven problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*10 pts*) Give an example of:

a. (*2 pts*) An open interval: ____________

b. (*2 pts*) A function that is *not* one-to-one: ____________

c. (*2 pts*) A power function: ____________

d. (*2 pts*) A rational number: ____________

e. (*2 pts*) A rational function: ____________

2. (*18 pts*) Fill in the blanks:

a. (*2 pts*) If $f(-2) = 0$, then $f^{-1}(0) =$ ____________ .

b. (*2 pts*) If $(4, 1)$ is a point on the graph of $f(x)$, then the point (___ , ___) is on the graph of $f(x - 2)$.

c. (*2 pts*) If $(2, -3)$ is a point on the graph of $f(x)$, and $f(x)$ is an odd function, then the point (___ , ___) is also on the graph of $f(x)$.

d. (*2 pts*) If $f(x) = 2 - 3x$, then a change of $\Delta x = 4$ will always result in a change of $\Delta y =$ ________ .

e. (*2 pts*) The statement "If x is rational, then x is negative" happens to be false; one counterexample is: ____________ .

f. (*2 pts*) Write this set using the simplest possible form of interval notation:

$$(-2, 3] \cap (-\infty, 1] = \underline{\hspace{4cm}} .$$

g. (*2 pts*) Write using the simplest possible form of interval notation:

$$(-2, 3] \cup (-\infty, 1] = \underline{\hspace{4cm}} .$$

h. (*4 pts*) Write down the cases you would need to consider in order to solve the inequality $x^2 - x - 2 > 0$. (Do **not** solve the inequality; just right down the cases.)

$$\Big(\underline{\hspace{2.5cm}} \text{ AND } \underline{\hspace{2.5cm}} \Big) \text{ OR } \Big(\underline{\hspace{2.5cm}} \text{ AND } \underline{\hspace{2.5cm}} \Big)$$

3. (*24 pts*) Show all work for each of the calculations below. (You may use your calculator to check your answers, but you must algebraically justify each of your answers.)

a. (*6 pts*) Find the equation of the linear function with slope 5 that passes through the point $(3, -2)$.

b. (*6 pts*) Find the domain of $f(x) = \dfrac{\sqrt{x-1}}{3x-5}$. Show all work clearly by hand.

c. (*6 pts*) Determine algebraically (**not** with a graph) whether the function $f(x) = \dfrac{x^3}{x^2+1}$ is even, odd, or neither.

d. (*6 pts*) Write $f(x) = |5 - 3x|$ as a piecewise function where each piece is defined on an *interval* of x-values.

4. (*6 pts*) Prove that the average rate of change of $f(x) = -2x + 4$ on *any* interval $[a, b]$ is always equal to -2. (Remember you are writing a *proof* here, so your argument must be clear!)

5. (*18 pts*) Give **short** answers to each of the questions below.

a. (*3 pts*) State the triangle inequality.

b. (*3 pts*) State the Induction Axiom.

c. (*3 pts*) State the converse of the statement "If $a < b$, then $a < b + 1$" (your statement in such a way that you do **not** use the word "not").

d. (*3 pts*) State the contrapositive of the statement "If $a < b$, then $a < b + 1$" (your statement in such a way that you do **not** use the word "not").

e. (*3 pts*) Complete this theorem: For any real numbers A and B,
$A \cdot B = 0$ if and only if:

f. (*3 pts*) Complete this theorem: For any real numbers A and B,
$\dfrac{A}{B} = 0$ if and only if:

6. (*9 pts*) Use the given values of $f(x)$ and $g(x)$ to complete the table below.

x	$f(x)$	$g(x)$	$(f-2g)(x)$	$(g\circ f)(x)$	$g^{-1}(x)$
1	3	2			
2	1	0			
3	2	3			

7. (*15 pts*) To the right of each part below, sketch the graph of a function $f(x)$ with domain $(-\infty,\infty)$ that has the listed properties. Make sure that the axes of your graph are clearly labeled.

a. (*3 pts*) $f(x)$ has a local maximum at $x=3$, a global maximum at $x=1$, and no global minimum.

b. (*3 pts*) $f(x)$ is negative and increasing everywhere, and has a horizontal asymptote at $y=-2$.

c. (*3 pts*) Given that $f(x)$ is the graph you drew in part **(b)** above, sketch a graph of $g(x)=f(-x)$.

d. (*3 pts*) $f(x)$ is concave up on $(-\infty,2)$ and concave down on $(2,\infty)$, always increasing, and $f(2)=-1$.

e. (*3 pts*) Given that $f(x)$ is the graph you drew in part **(d)** above, sketch a graph of $g(x)=f(x+2)-3$.

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK