TEST II

Math 231
March 25, 2003

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for. Draw a fish on the scrap page for two points.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 9 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*16 pts*) Determine whether each of the following statements is true (T) or false (F).

a. (*2 pts*) **T** **F** If f is continuous on $[2, 5]$, then f is continuous for all $x \in [2, 5]$.

b. (*2 pts*) **T** **F** The average rate of change of $f(x) = 9 - x^2$ on $[-1, 2]$ is negative.

c. (*2 pts*) **T** **F** If f changes sign at $x = 2$ and $f(2) \neq 0$, then f is discontinuous at $x = 2$.

d. (*2 pts*) **T** **F** If f is discontinuous at $x = 3$, then f is not differentiable at $x = 3$.

e. (*2 pts*) **T** **F** If f has a critical point at $x = 1$, then f has a local extrema at $x = 1$.

f. (*2 pts*) **T** **F** If f is continuous everywhere, and if $f(-2) = 3$ and $f(1) = 2$, then f must have a root somewhere in $(-2, 1)$.

g. (*2 pts*) **T** **F** If f is continuous everywhere, and if $f(1) = -2$ and $f(4) = 3$, then f must have a root somewhere in $(1, 4)$.

h. (*2 pts*) **T** **F** If f is continuous and differentiable everywhere, and if $f(2) = 0$ and $f(8) = 0$, then f' must have a root somewhere in $(2, 8)$.

2. (*12 pts*) Give precise mathematical statements of the following.

a. (*3 pts*) The constant multiple rule for limits.

b. (*3 pts*) The constant multiple rule for derivatives.

c. (*3 pts*) The Extreme Value Theorem.

d. (*3 pts*) The Mean Value Theorem.

3. (*16 pts*) Show that $\frac{d}{dx}(x^2) = 2x$ without using any "differentiation rules," as follows.

a. (*8 pts*) Using the definition of derivative.

b. (*8 pts*) Using the alternate definition of derivative.

4. (*12 pts*) Use differentiation rules (*not* the definition) to calculate the following derivatives.

a. (*6 pts*) If $f(x) = \dfrac{x^4 - 7x^3}{2x}$, find $f'(x)$.

b. (*6 pts*) If $f(x) = x - 3x^5$, find $\left.\dfrac{d^3 f}{dx^3}\right|_2$.

5. (*8 pts*) Use the definition of derivative to prove the sum rule for differentiation (*i.e.* to prove that $(f(x) + g(x))' = f'(x) + g'(x)$). Justify each step in your proof.

6. (*8 pts*) Use the axes below to sketch graphs of Stuart's position and velocity as described in the following paragraph.

> *Stuart left his house at noon and walked north on Pine Street for twenty minutes. At this point he realized he was late for an appointment at the dentist, whose office was located* south *of Stuart's house on Pine Street; fearing he would be late, Stuart sprinted south on Pine Street, past his house, and to the dentist's office. When he got there, he found the office closed for lunch; he was 10 minutes early for his 12:40 appointment. Stuart waited at the office for 10 minutes and then found out that his appointment was actually for tomorrow, so he then walked back to his house.*

position velocity

7. (*8 pts*) Find the largest value of δ (accurate to three decimal places) for which the following statement is true:

$$0 < |x - 2| < \delta \implies |x^3 - 8| < 0.5$$

Show all work, including a sketch of $f(x) = x^3$ that illustrates the roles of 2, 8, 0.5, and δ.

8. (*12 pts*) Calculate the following limits of $f(x) = \frac{x}{x^2 - x}$, showing all work. If a limit does not exist, find the left and right limits.

a. (*6 pts*) $\lim_{x \to 0} \frac{x}{x^2 - x}$.

b. (*6 pts*) $\lim_{x \to 1} \frac{x}{x^2 - x}$.

9. (*8 pts*) Circle <u>all</u> of the following that pertain to the function $f(x) = \begin{cases} 3x + 2, & \text{if } x \leq 2 \\ x^3, & \text{if } x > 2. \end{cases}$

(A) $f(x)$ is left continuous at $x = 2$.

(B) $f(x)$ is right continuous at $x = 2$.

(C) $f(x)$ is differentiable at $x = 2$.

(D) $f(x)$ is right differentiable at $x = 2$.

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK