TEST II

Math 231
October 26, 2004

Name: ______________________________

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 6 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*16 pts*) Determine whether each of the following statements is true (T) or false (F).

(a) **T** **F** If $x = c$ is a critical point of f, then f has either a local maximum or a local minimum at $x = c$.

(b) **T** **F** The instantaneous rate of change of a function $f(x)$ at a point $x = c$ is equal to the slope of the line tangent to $f(x)$ at $x = c$.

(c) **T** **F** $\lim_{x \to c} f(x)$ is always equal to $f(c)$.

(d) **T** **F** $\lim_{x \to c} f(x)$ is always equal to $f'(c)$.

(e) **T** **F** If f is not differentiable at $x = c$, then f is not continuous at $x = c$.

(f) **T** **F** f is continuous at $x = c$ if and only if f is both left and right continuous.

(g) **T** **F** f is differentiable at $x = c$ if and only if f is both left and right differentiable.

(h) **T** **F** I would like two free points, please.

2. (*12 pts*) For each part below, give a formal definition of the given limit statement (your answer may involve δ, ϵ, M, or N). Then sketch and label a graph that illustrates your definition.

(a) $\lim_{x \to 2} (x^2 + 1) = 5$

(b) $\lim_{x \to \infty} \frac{1}{x} = 0$

3. (*25 pts*) Show all steps and all work carefully to calculate each of the following. You will be graded on the quality and clarity of your work, not just your final answers. Work by calculator is not sufficient. Please circle your final answers.

(a) Find $\displaystyle\lim_{x\to 1} \frac{1}{1-x}$.

(b) Find $\displaystyle\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$.

(c) Use the definition of derivative to find $\frac{d}{dx}\left(\frac{1}{x}\right)$.

(d) If $f(x) = \begin{cases} x^2, & \text{if } x \le 1 \\ 2x+3, & \text{if } x > 1 \end{cases}$, find $f'_+(1)$.

(e) If $f(x) = 2x^3(3x+1)$, find $\displaystyle\left.\frac{d^2 f}{dx^2}\right|_{x=1}$.

(You MAY use differentiation rules/shortcuts we proved in class.)

4. (*25 pts*) Give precise mathematical statements for the following definitions and theorems.

(a) State the sum rule for limits.

(b) Define in terms of limits what it means for a function f to have a jump discontinuity at $x = 4$.

(c) Define in terms of limits what it means for a function f to be left-differentiable at a point $x = c$.

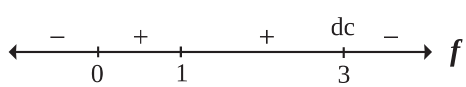
(d) State the Intermediate Value Theorem.

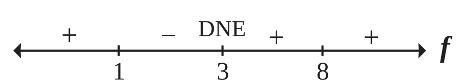
(e) State Rolle's Theorem, and sketch a graph illustrating the theorem.

5. (*10 pts*) Use the alternate definition of derivative to prove that $\frac{d}{dx}(x^6) = 6x^5$. You will need a factoring formula.

6. (*12 pts*) For each part below, sketch the graph of a function f that has the given number line. The symbol "dc" means "discontinuous" and the symbol "DNE" means "does not exist."

Notice that in part (a) you are given a number line for f, while in part (b) you are given a number line for f'. In BOTH parts you are to draw a possible graph of the function f (<u>not</u> f'). The two parts of this problem are NOT related to each other. Don't make your graph too small, and be sure the features of your graph are clear.

(**a**) 

(**b**) 

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK