TEST III

Math 231
November 23, 2004

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 6 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. **(*14 pts*)** Determine whether each of the following statements is true (T) or false (F).

(**a**) **T** **F** A power function $f(x) = Ax^k$ is differentiable everywhere on its domain if $k \geq 0$.

(**b**) **T** **F** A power function $f(x) = Ax^{\frac{p}{q}}$ is an even function if p and q are both even.

(**c**) **T** **F** If $f(x) = x^{\frac{p}{q}}$ has an inverse, then its inverse is the function $f^{-1}(x) = x^{-\frac{p}{q}}$.

(**d**) **T** **F** If $f''(2) = 0$, then $x = 2$ is an inflection point of f.

(**e**) **T** **F** If f'' is positive for all $x \in (-1, 1)$, then f' is increasing for all $x \in (-1, 1)$.

(**f**) **T** **F** If f is a polynomial function with constant term $a_0 = 3$, then $x = 3$ is a root of f.

(**g**) **T** **F** Math is cool.

2. **(*12 pts*)** Give short answers to each of the following.

(**a**) Show that the polynomial function $f(x) = x^2 + 3x + 10$ is irreducible.

(**b**) Write down an equation for a six-degree polynomial function that has only two distinct roots. *(There is more than one possible answer.)*

(**c**) If there is a nonzero remainder R after using synthetic division to divide a polynomial function f by a linear expression $x - c$, then we can write f in terms of $(x - c)$, R, and some polynomial function $g(x)$ as follows:

$$f(x) = \_\_\_\_\_ .$$

3. (*24 pts*) Show all work in each of the following calculations. (For parts (c) and (d) you may use any differentiation rules/shortcuts that we have learned in this class.)

(**a**) Find $\lim_{x\to\infty} \dfrac{1}{1-2x^{\frac{1}{7}}}$.

(**b**) Find $\lim_{x\to\infty} (x^{\frac{3}{2}} - x^5)$.

(**c**) Find the derivative of $f(x) = \dfrac{5x}{(2\sqrt[3]{x})^4}$.

(**d**) Find the derivative of $f(x) = (2x^6 + x^{-2})(x^4 - 3x^{\frac{1}{5}})$.

4. (*30 pts*) Fill in the blanks or circle answers as appropriate. You do NOT have to show work, but a little bit of work might earn you some partial credit if you get an answer wrong.

(**a**) If $f(x) = \dfrac{x^{\frac{1}{4}}}{\sqrt{x}(4x^{-3})^2}$ is written in the form $f(x) = Ax^k$, what are A and k?

$A =$ _____

$k =$ _____

(**b**) If $f(x)$ is the polynomial function $f(x) = 3x^2 - 5x + x^4 + 6$, then:

degree = _____ $a_1 =$ _____ $a_3 =$ _____

The possible integer roots of f are: _____

(**c**) Suppose that $f(x) = \dfrac{x^4 - 2x^3 + x^2}{x^2 - 1}$.

List the locations of any roots of $f(x)$: _____

List the locations of any "holes" in the graph of $f(x)$: _____

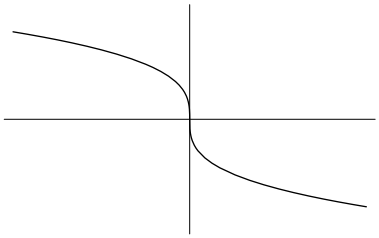
(**d**) Fill in the blanks with numbers a and b that will make the piecewise function $f(x)$ given below continuous and differentiable at $x = 1$.

$$f(x) = \begin{cases} 3x + a, & x < 1 \\ bx^2 + 1, & x \geq 1 \end{cases}$$

$a =$ _____

$b =$ _____

(**e**) If $f(x) = Ax^{\frac{p}{q}}$ is a power function whose graph has the shape shown below, what can you say about A, p, and q? (Circle ***all*** properties that ***must*** be true.)



A is (POS) (NEG) (EVEN) (ODD)

p is (EVEN) (ODD)

q is (EVEN) (ODD)

$\frac{p}{q}$ is (POS) (NEG) (LESS THAN 1) (MORE THAN 1)

5. (*8 pts*) Use the second derivative test to show that $x = 2$ is a local minimum of the function $f(x) = x^3 - 12x + 1$. Be sure that your argument is clear.

6. (*12 pts*) Farmer Calvin wants to build a rectangular ostrich pen along the side of a river (so only three sides of fence are needed). What is the largest such pen that Farmer Calvin can enclose with 540 feet of fencing material? (Show all work clearly and in order, and be sure that you explain how you know you have found the pen with the largest area.)

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK