TEST I

Math 231
September 22, 2005

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You may not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*8 pts*) Determine whether each of the following statements is true (T) or false (F). You do not have to provide reasons or counterexamples.

(**a**) **T** **F** If $c < 0$, then $|c| = -c$.

(**b**) **T** **F** If $\frac{5}{x+1} < 1$, then $5 < x + 1$.

(**c**) **T** **F** For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that $x < y$.

(**d**) **T** **F** There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$ we have $x < y$.

2. (*6 pts*) Fill in the blanks.

(**a**) If $(1, -6)$ is on the graph of $f(x)$, then (___ , ___) is on the graph of $f(x + 2) + 3$.

(**b**) If $(3, 2)$ is on the graph of $f(x)$, then (___ , ___) is on the graph of $f(3x)$.

(**c**) If $f(x)$ is an odd function and $(2, 5)$ is on the graph of $f(x)$, then (___ , ___) is also on the graph of $f(x)$.

3. (*12 pts*) Give precice mathematical definitions (NOT intuitive descriptions) of each of the following. Please, no rambling or long sentences; just simple mathematical definitons.

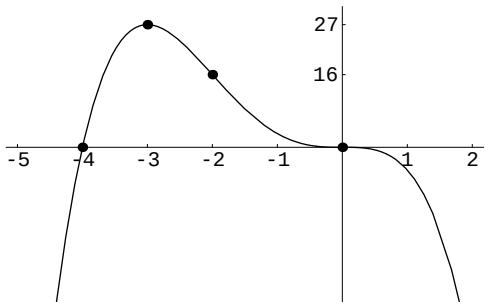
(**a**) If A and B are sets, then f is a *function* from A to B if:

(**b**) A function f has a *global maximum* at $x = c$ if:

(**c**) A function f is said to be *increasing* on an interval I if:

4. **(*10 pts*)** Find the domain of $f(x) = \dfrac{\sqrt{x-3}}{x^2-6x+5}$. Show your work clearly and in order so that I can see how you arrived at your answer. Write your answer in interval notation.

5. **(*18 pts*)** Fill in each of the blanks with intervals or points, as appropriate, given that $f(x)$ has the graph below. Note that you may have to approximate some values from the graph.



(**a**) _____ The interval(s) on which $f(x)$ is negative.

(**b**) _____ The interval(s) on which $f(x)$ is increasing.

(**c**) _____ The interval(s) on which $f(x)$ is concave down.

(**d**) _____ The location(s) of any roots of $f(x)$.

(**e**) _____ The location(s) of any local maxima of $f(x)$.

(**f**) _____ The location(s) of any inflection points of $f(x)$.

6. **(*24 pts*)** Short answers. No work required.

(**a**) Express the sentence "The distance between y and -8 is less than three" as an absolute value inequality.

(**b**) Write the solution to $|x-4| \geq 0.5$ in interval notation. (Hint: It may help to draw something on a number line first.)

(**c**) The following statement is *false*: "If $x > 0$, then $x^2 > x$." Write down a counterexample and briefly say why it is a counterexample.

(**d**) Write the range of the function $f(x) = 6 - x^2$ in interval notation.

(**e**) Give an example of a function $f(x)$ that is a power function but is not a polynomial function.

(**f**) Write the function $f(x) = |3x + 1|$ as a piecewise function (where each piece is defined on an interval of x-values).

7. (*12 pts*) Suppose A, B, and C represent logical statements. Simplify each of your answers below, if possible.

(**a**) The converse of "$A \Longrightarrow$ Not B" is:

(**b**) The contrapositive of "$C \Longrightarrow B$" is:

(**c**) The negation of "B and Not C" is:

8. (*10 pts*) Prove ***ONLY ONE*** of the following statements.

(**a**) The sum of any two rational numbers is rational.
(Justify all relevant steps and make sure that your argument is clear.)

(**b**) For all positive integers n, we have $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.
(Use induction.)

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK