TEST II

Math 231
October 27, 2005

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You may not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 7 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*16 pts*) Determine whether each of the following statements is true (T) or false (F). You do not have to provide reasons or counterexamples.

(**a**) **T** **F** If f^{-1} is the inverse of a function f, then $f^{-1}(b) = a$ if and only if $f(a) = b$.

(**b**) **T** **F** If f is an even function whose domain is $(-\infty, \infty)$, then f cannot have an inverse.

(**c**) **T** **F** If f is both left-continuous at $x = 3$ and right-continuous at $x = 3$, then f is continuous at $x = 3$.

(**d**) **T** **F** If f is both left-differentiable at $x = 3$ and right-differentiable at $x = 3$, then f is differentiable at $x = 3$.

(**e**) **T** **F** $\dfrac{d}{dx}(2(1 + 3x^2)) = 12x$.

(**f**) **T** **F** The tangent line to $f(x)$ at $x = 3$, if one exists, is the line through $(3, f(3))$ with slope $f'(3)$.

(**g**) **T** **F** If $0 < |x + 2| < 0.1$, then $|x^2 - 4| < 0.4$.

(**h**) **T** **F** I would like two free points, please.

2. (*12 pts*) Give precice mathematical definitions (NOT intuitive descriptions) of each of the following. (Hint: Each of your answers should involve limits somehow!)

(**a**) A function f is *continuous* at $x = c$ if:

(**b**) A function f has a *removable discontinuity* at $x = c$ if:

(**c**) A function f is *right-differentiable* at $x = c$ if:

3. (*10 pts*) Use the definition of derivative to show that $\frac{d}{dx}(7x^2 + 1) = 14x$.
(Do NOT use the "shortcut rules," use the definition or the alternate definition of derivative.)

4. (*10 pts*) Write a delta-epsilon proof for the limit statement $\lim_{x \to 2}(3 - 4x) = -5$.

5. **(*24 pts*)** Give short answers or circle answers as indicated. No work required.

(**a**) State the product rule for limits.

(**b**) State the constant multiple rule for derivatives.

(**c**) State the Extreme Value Theorem.

(**d**) State Rolle's Theorem.

(**e**) If $f(x) = x^3$, circle **ALL** of the following that are equivalent to $f'(x)$.

A. $\dfrac{df}{dx}$

B. $\lim\limits_{w \to x} \frac{5w^3 - 5x^3}{w - x}$

C. $5\,\dfrac{d}{dx}(x^3)$

D. $\lim\limits_{x \to 0} 5x^3$

E. $\dfrac{df}{dx}(5x^3)$

F. $\lim\limits_{x \to 2} \frac{5(2+h)^3 - 5(2)^3}{h}$

G. $\lim\limits_{h \to 0} \frac{5(x+h)^3 - 5x^3}{h}$

H. $15x^2$

I. $\dfrac{d}{dx}(5x^3)$

(**f**) Given that $\lim\limits_{x \to 2^-} f(x) = 2$, $\lim\limits_{x \to 2^+} f(x) = 1$, and $f(2) = 1$, circle **ALL** the statements that are true.

A. $\lim\limits_{x \to 2} f(x)$ exists

B. f is continuous at $x = 2$

C. f is left-continuous at $x = 2$

D. f is right-continuous at $x = 2$

E. f has a jump discontinuity at $x = 2$

F. f has a removable discontinuity at $x = 2$

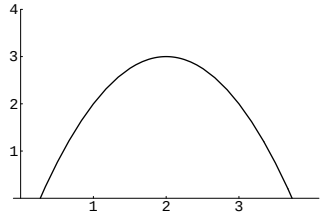
6. (*16 pts*) Calculate the following. Show all work.

(**a**) $\lim_{x \to 2} \dfrac{3}{2 - x}$

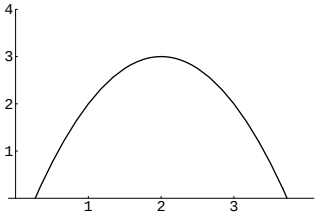
(**b**) $f'_+(1)$, where $f(x) = \begin{cases} x^2, & \text{if } x \leq 1, \\ 2x + 4, & \text{if } x > 1. \end{cases}$

7. (*12 pts*) The graph of a function $f(x)$ is shown six times in the graphs below. On each graph, sketch a line with the indicated property.

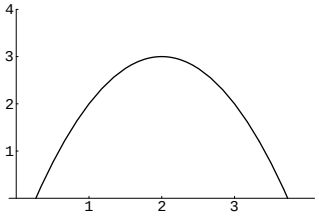
The secant line from $(2, f(2))$ to $(2, f(2.5))$



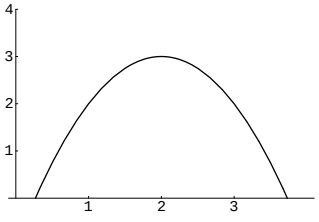
The tangent line to $f(x)$ at $x = 1.5$



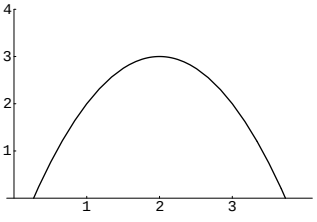
Line whose slope is equal to $\frac{f(2.5)-f(1)}{2.5-1}$



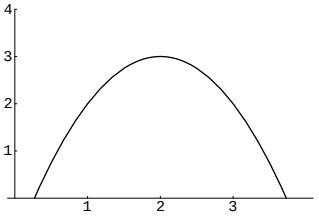
Line whose slope is equal to $f'(3)$



Line whose slope is $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$



Line whose slope is $\lim_{x \to 1} \frac{f(x)-f(1)}{x-1}$



Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK