TEST III

Math 231
December 1, 2005

Name: _____
By writing my name I swear by the honor code.

Calculators are NOT allowed on this exam.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You may not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- By writing your name above, you agree to the JMU honor code. In particular, this means that you may not use any notes or crib sheets during this exam, that all work must be your own, and that you may not obtain advance information revealing the problems on this exam.
- This test has 6 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*12 pts*) Determine whether each of the following statements is true (T) or false (F). You do not have to provide reasons or counterexamples.

(**a**) **T** **F** If $x = 2$ is a minimum of the derivative f' of some function f, then f has an inflection point at $x = 2$.

(**b**) **T** **F** If $f'(1) = 0$ and $f''(1) = -2$, then $x = 1$ is a local maximum of f.

(**c**) **T** **F** $\dfrac{d}{dx}\left(\dfrac{4}{\sqrt{x^5}}\right) = \dfrac{-10}{\sqrt{x^7}}$.

(**d**) **T** **F** If $f(x) = x^{\frac{5}{3}}$, then $f^{-1}(x) = x^{-\frac{5}{3}}$.

(**e**) **T** **F** There is no six-degree polynomial with exactly three distinct roots, one of which is a triple root and two of which are single roots.

(**f**) **T** **F** If f is a seven-degree polynomial with exactly four distinct roots, one of which is a double root, then f must have exactly four turning points.

2. (*18 pts*) Calculate the following limits algebraically. Show all work so I can see how you arrived at your answer.

(**a**) $\lim\limits_{x\to 0}(x^{-2} - x^{-1})$

(**b**) $\lim\limits_{x\to \infty}(-2x^5 + 8x^4 - 6)$

(**c**) $\lim\limits_{x\to \infty}\dfrac{x-2}{1+3x}$

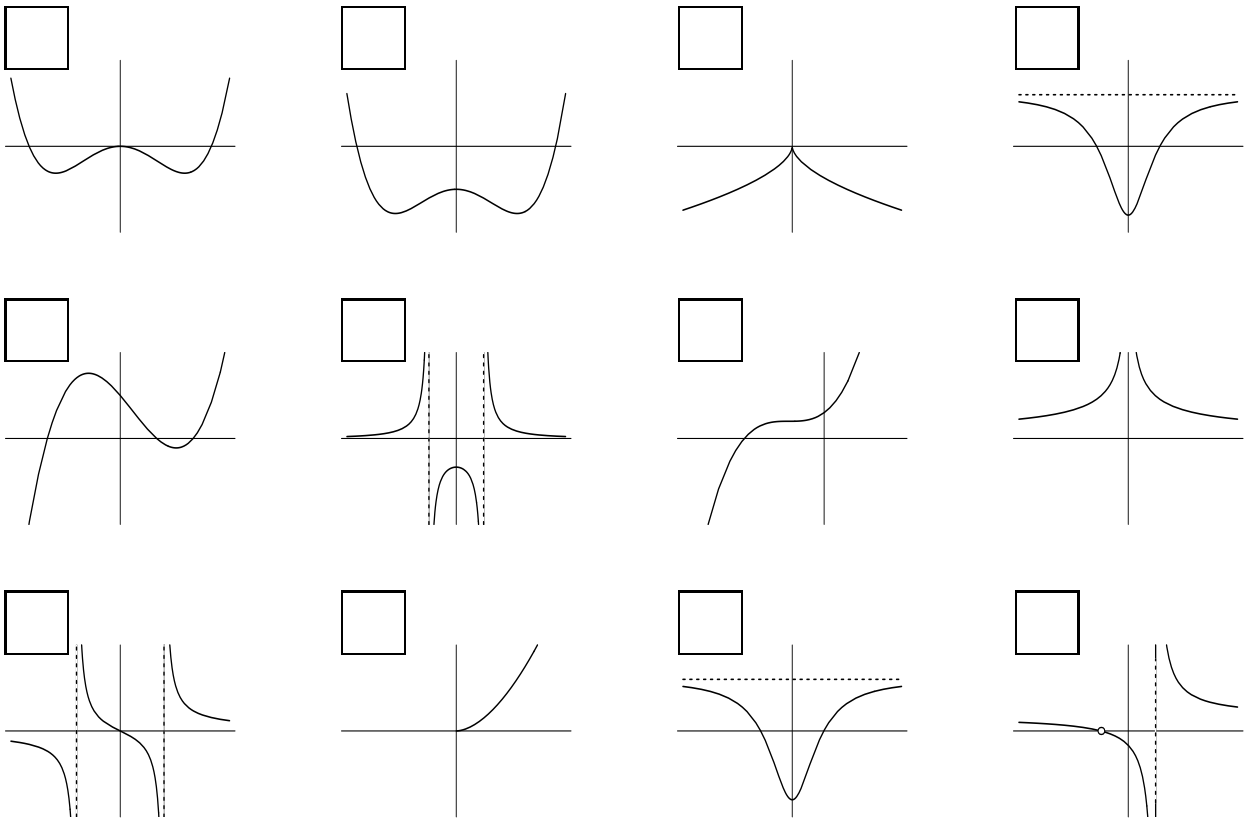
3. **(*24 pts*)** Show your work clearly in each of the following calculations.

(**a**) Factor $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$ as much as possible.

(**b**) Find the equation of the slant asymptote of $f(x) = \dfrac{x^3 + x^2 - 6x}{x^2 - 4x + 4}$.

(**c**) Use the definition of derivative (NOT the power rule) to find the derivative of $f(x) = x^{\frac{1}{2}}$.

4. (*24 pts*) Match each graph with one of the functions listed at the bottom of the page, by putting the letter for the function into the box above the corresponding graph. (Note: The scale/graphing window is different for each graph.)



A. $f(x) = -x^{\frac{4}{7}}$

B. $f(x) = x^{-\frac{4}{7}}$

C. $f(x) = x^{\frac{7}{4}}$

D. $f(x) = x^{\frac{2}{5}}$

E. $f(x) = -2x^{\frac{2}{5}}$

F. $f(x) = x^{\frac{2}{3}} - 3$

G. $f(x) = (x+1)^3 + 2$

H. $f(x) = (x-1)(x+1)$

I. $f(x) = x(x-1)(x+1)$

J. $f(x) = x^2(x^2-1)$

K. $f(x) = (x^2-4)(x^2+1)$

L. $f(x) = (x+2)(x-2)(x-5)$

M. $f(x) = x^3 - 3x + 2$

N. $f(x) = x^3 - x^2 - 4x + 4$

O. $f(x) = \dfrac{x+1}{x-2}$

P. $f(x) = \dfrac{x+1}{(x+1)(x-2)}$

Q. $f(x) = \dfrac{x+1}{(x+1)^2(x-2)}$

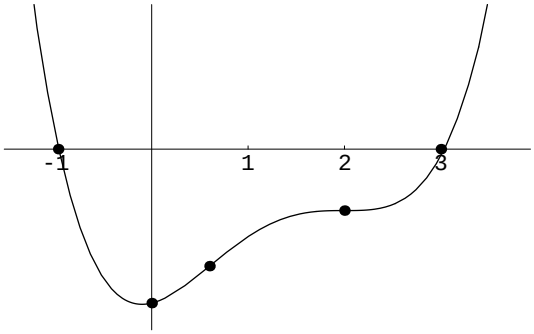
R. $f(x) = \dfrac{(x+1)^2}{(x+1)(x-2)}$

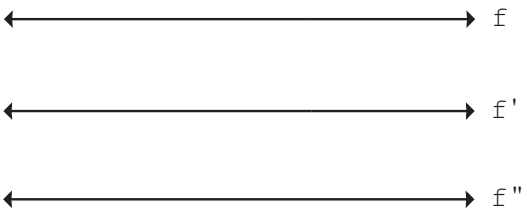
S. $f(x) = \dfrac{x-1}{x(x-1)}$

T. $f(x) = \dfrac{3x^2-4}{x^2+1}$

U. $f(x) = \dfrac{x}{x^2-4}$

5. (*12 pts*) Use the graphical information in the given graph of f to fill in the three number lines on the right.





6. (*10 pts*) Set up, state, and solve an optimization problem to find the area of the largest possible rectangular cow pasture that can be fenced off with 1000 feet of fencing material.

Diagram and setting up:

State the problem:
Find the global maximum of the function _____ *on the interval* _____ .

Solve the problem:

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK