CHAPTER 4 TEST

All brains and no machines, please.

Math 231
October 22, 2008

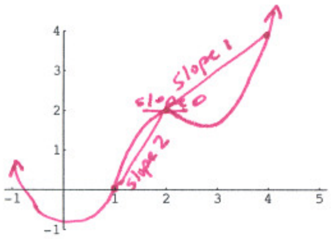
Name: * Key *

By printing my name I pledge to uphold the honor code.

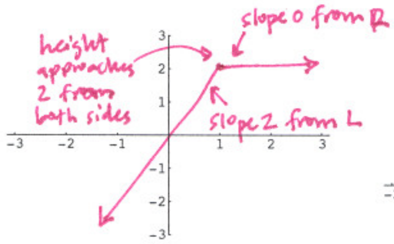
HW 3.2 #10 3.3 #4 3.8 #55

1. For each set of axes, sketch a graph of f with all the properties listed below it.

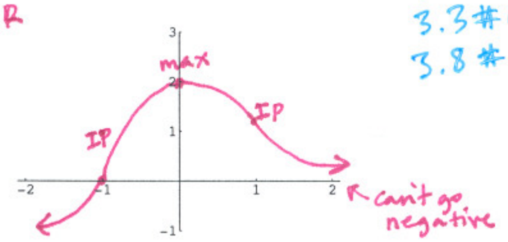
9 pts each graph → 27 pts

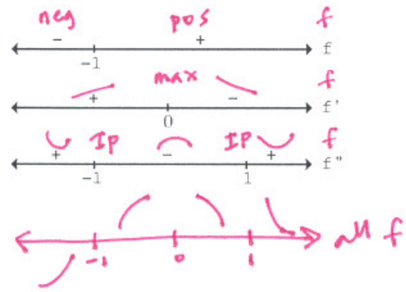


- $\frac{f(2)-f(1)}{1} = 2$
- $\frac{f(4)-f(2)}{2} = 1$
- $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 0$



- $\lim_{x \to 1} f(x) = 2$
- $\lim_{h \to 0^-} \frac{f(1+h)-f(1)}{h} = 2$
- $\lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h} = 0$





2. **a)** What does Rolle's Theorem say about $f(x) = x^2 - 3x - 4$ on the interval $[-1, 4]$?

HW 3.6 #50

$f(x) = (x+1)(x-4)$

since f is cont. on $[0,4]$ and diff. on $(0,4)$ (it's a polynomial),
and $f(-1) = f(4) = 0$,
RT says that there is some $c \in (-1,4)$ where $f'(c) = 0$.

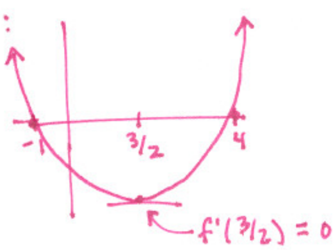
12 pts each part → 24 pts

b) In your answer above you should have said something "exists." Use calculus to find it. (You MAY use derivative rules/shortcuts here.)

find a $c \in (-1,4)$ where $f'(c) = 0$:

$f'(x) = 2x - 3 = 0 \Rightarrow 2x = 3 \Rightarrow x = 3/2$.

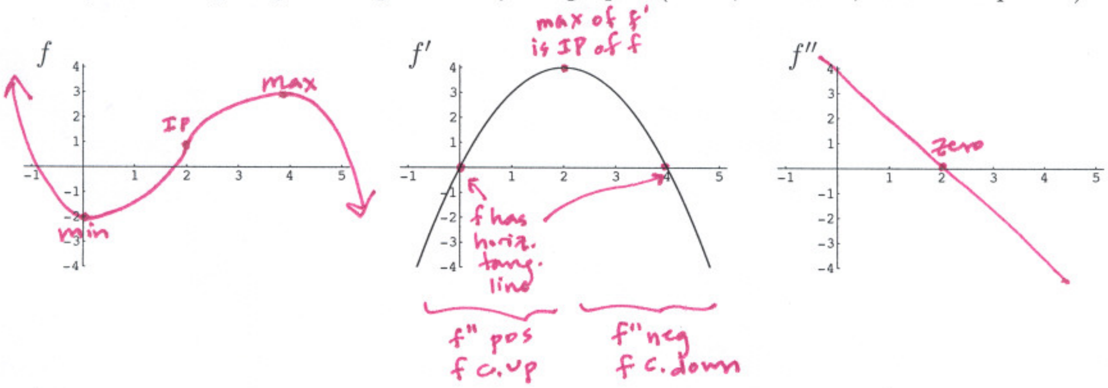
in case you're interested, here is the graph:



f' neg f dec   f' pos so f inc.   f' neg f dec

3. Given the graph of f' shown below, sketch possible graphs of f and f''. Be sure to clearly mark any important points of your graphs (zeros, extrema, inflection points). HW 3.7 #22 HW 3.8 #69

12 pts each graph → 24 pts



4. **a)** Use the definition of derivative to find the derivative of $f(x) = x^2 - 4$. Show your work clearly. (Do NOT use the shortcuts/rules. We know the answer will be $2x$.) HW 3.4 #38 HW 3.1 #51, 60

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{((x+h)^2-4)-(x^2-4)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-\cancel{4}-x^2+\cancel{4}}{h} = \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0} (2x+h) = 2x+0 = \underline{\underline{2x}}.$$

12 pts each part → 24 pts

b) Now do the same calculation but using the *alternative* definition of derivative.

$$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{(z^2-4)-(x^2-4)}{z-x}$$

$$= \lim_{z\to x} \frac{z^2-\cancel{4}-x^2+\cancel{4}}{z-x} = \lim_{z\to x} \frac{(z+x)\cancel{(z-x)}}{\cancel{z-x}}$$

$$= \lim_{z\to x} (z+x) = x+x = \underline{\underline{2x}}.$$

+1 free

total poss. is 100