* scale up to 150 pts

CHAPTERS 4 AND 5 TEST

No calculators or cell phones. Good luck!

Math 231
November 17, 2008

Name: * key *

By printing my name I pledge to uphold the honor code.

1. Calculate each of the following. Show all work and circle your final answers. (For the differentiation problem, you MAY use differentiation rules we have covered in class.)

2 only to $\sqrt[3]{\infty} - \infty$
4 only to $\infty - \infty$
6 successful factoring
6 $\infty - \infty$ intuition

a) $\lim_{x\to\infty} (x^{\frac{1}{3}} - x) \to \sqrt[3]{\infty} - \infty \to \infty - \infty \to$?? indet.

10 pts ↳ $= \lim_{x\to\infty} x^{1/3}(1 - x^{2/3}) \to \sqrt[3]{\infty}(1 - \sqrt[3]{\infty^2}) \to \infty(-\infty) \to \boxed{-\infty}$.

(also could factor out x)

6 tried conj.
1 @ plug in, 0/0
½ valiant but no

b) $\lim_{h\to 0} \frac{\sqrt{3+h} - \sqrt{3}}{h} \left(\frac{\sqrt{3+h} + \sqrt{3}}{\sqrt{3+h} + \sqrt{3}}\right) = \lim_{h\to 0} \frac{(3+h) - 3}{h(\sqrt{3+h} + \sqrt{3})}$

10 pts ↳ $= \lim_{h\to 0} \frac{h}{h(\sqrt{3+h} + \sqrt{3})} = \lim_{h\to 0} \frac{1}{\sqrt{3+h} + \sqrt{3}} = \frac{1}{\sqrt{3+0} + \sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}}$.

3 pts alg only to clear denom.
1-2 pt bad deriv right away
5 pts bad alg but good deriv & reasonable

c) $\frac{d}{dx}\left(\frac{\sqrt{x} + 3x^5}{x^{-1}}\right) = \frac{d}{dx}\left(\frac{\sqrt{x} + 3x^5}{1/x} \cdot \frac{x}{x}\right) = \frac{d}{dx}\left(\frac{x^{3/2} + 3x^6}{1}\right)$

10 pts ↳ $= \boxed{\frac{3}{2}x^{1/2} + 18x^5}$.

2. Joe the Farmer wants to enclose his chickens in three equal-sized side-by-side pens using 1000 feet of fencing as shown in the figure. What is the math problem that you would have to solve to tell Joe the fencing dimensions that would allow the largest chicken pens? Show any relevant work in translating this problem.

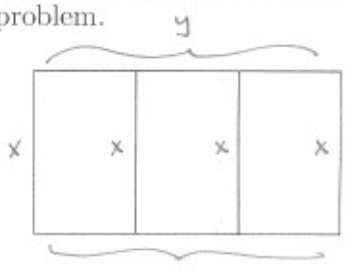
given: $4x + 2y = 1000 \Rightarrow y = \frac{1000 - 4x}{2} = 500 - 2x$

find: max area $A = xy = x(500 - 2x)$

14 pts

smallest x: 0

smallest y: 0



$4x = 1000 \Rightarrow x = 250$

2 pts perimeter formula

DON'T SOLVE THE PROBLEM, JUST FILL IN THESE BLANKS:

Find the global maximum of the one-variable function $A(x) = x(500 - 2x)$ on the interval $[0, 250]$.

10 pts

4 pts

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3. Find a function f that has derivative $f'(x) = (x^4 - 8)(1 - 3x^5)$ and value $f(0) = 2$.

14 pts

$f'(x) = x^4 - 3x^9 - 8 + 24x^5$

so $f(x) = \frac{1}{5}x^5 - \frac{3}{10}x^{10} - 8x + \frac{24}{6}x^6 + C$

~~remainder~~

$f(0) = 2 \Rightarrow C = 2$

so $\boxed{f(x) = \frac{-3}{10}x^{10} + 4x^6 + \frac{1}{5}x^5 + 2}$.

+4 pts mult. out
+6 pts anti-der.
+4 pts "+2"

4. For $f(x) = x^3 - 2x^2 + x - 2$, make number lines decorated with $+/-$ as appropriate for f, f', and f''.

8 pts each (24 pts)

$f(x) = x^2(x-2) + (x-2)$

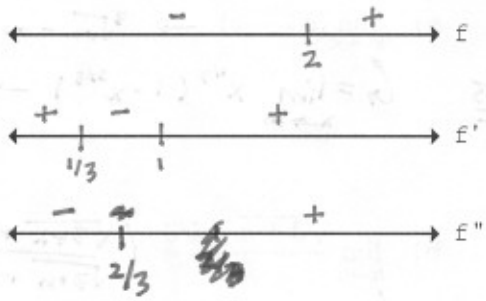
$f' = (x^2+1)(x-2)$ root $x=2$

$f'(x) = 3x^2 - 4x + 1$

$f' = (3x-1)(x-1)$ roots $x = \frac{1}{3}, x = 1$

$f''(x) = 6x - 4$

$f'' = 2(3x-2)$ root $x = 2/3$



2 pts A-D right away
2 pts some shred of something

each line:
4 pts roots, 4 pts +/−

5. Fill in the blanks with letters to match each description with one of the given functions.

A) $f(x) = 2x^{\frac{3}{4}} = 2(\sqrt[4]{x})^3$

B) $f(x) = 3x^{\frac{4}{3}} = 3(\sqrt[3]{x})^4$

C) $f(x) = -4x^{\frac{5}{3}} = -4(\sqrt[3]{x})^5$

D) $f(x) = 5x^{-\frac{5}{3}} = \frac{5}{(\sqrt[3]{x})^5}$

E) $f(x) = x^3 - 3x^2 + 4x - 12 \overset{*}{=} (x-3)(x^2+4)$

F) $f(x) = x^3 - 3x^2 - 4x + 12 \overset{**}{=} (x-3)(x^2-4) = (x-3)(x+2)(x-2)$

G) $f(x) = x^3 - 3x^2 - 9x + 27$
$= x^2(x-3) - 9(x-3) = (x^2-9)(x-3)$
$= (x-3)^2(x+3)$

H) not possible

3 pts each (18 pts)

E A function with only one real root but two turning points.

H A function with no real roots but two turning points.

A A function that is not defined for $x < 0$.

D A function with at least one asymptote.

G A function with at least one double root.

B A function whose graph has a non-differentiable cusp.

* $x=3$ is a root

	1	−3	4	−12
3		3	0	12
	1	0	4	0

** $x=3$ is a root

	1	−3	−4	+12
3		3	0	−12
	1	0	−4	0

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