CHAPTER 6 TEST

Your brain, your pen, your test. No calculators, phones, or looking around.

Math 231
December 3, 2008

Name: * Key *

By printing my name I pledge to uphold the honor code.

1. This entire exam is about the rational function $f(x) = \dfrac{x^2}{x-1}$.
We'll start with some easy facts about this function.

3 each
18 pts

a) $f(x)$ is a rational function because: it is a quotient of polynomials

b) In interval notation, the domain of $f(x)$ is: $(-\infty, 1) \cup (1, \infty)$

c) List any roots of $f(x)$: $x = 0$

d) List any holes of $f(x)$: ϕ

e) List any vertical asymptotes of $f(x)$: $x = 1$

f) List any horizontal asymptotes of $f(x)$: ϕ

2. Calculate the following limits. Show all work and algebra.

5 each
20 pts

3 pts $\frac{1}{0} \to$ correct answer but no $0^+, 0^-$
2 pts correct but no work
1 pt $\frac{1}{0}$ only

a) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x^2}{x-1} \to \frac{1}{1^+ - 1} \to \frac{1}{0^+} \to \boxed{\infty}$

b) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2}{x-1} \to \frac{1}{1^- - 1} \to \frac{1}{0^-} \to \boxed{-\infty}$

c) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2}{x-1} \frac{(1/x^2)}{(1/x^2)} = \lim_{x \to \infty} \frac{1}{1/x - 1/x^2} = \lim_{x \to \infty} \frac{1}{1/x(1-1/x)} \to \frac{1}{0^+(1)} \to \boxed{\infty}$

d) $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2}{x-1} \frac{(1/x^2)}{(1/x^2)} = \lim_{x \to \infty} \frac{1}{1/x - 1/x^2} = \lim_{x \to \infty} \frac{1}{1/x(1-1/x)} \to \frac{1}{0^-(1)} \to \boxed{-\infty}$

3. $f(x)$ has a slant asymptote. Find its equation. Make sure I can tell how you found it.

10 pts

$$\begin{array}{r|ccc} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & \boxed{1} \end{array}$$

OR

$$\begin{array}{r} x+1 \\ x-1 \overline{)\, x^2+0x+0} \\ \ominus x^2 \oplus x \\ \hline x+0 \\ \ominus x \oplus 1 \\ \hline 1 \end{array}$$

$\Rightarrow x^2 = (x-1)(x+1) + 1$

$\Rightarrow f(x) = \dfrac{x^2}{x-1} = (x+1) + \dfrac{1}{x-1}$

$\boxed{y = x+1}$ is eqn of slant asymptote

8 pts work
2 pts for knowing which part is the asy

-3 stopping at x
w/ 2 pts overall $\frac{x^2}{x} = x$

3 pts $\frac{\infty}{\infty}$ & correct
1 pt $\frac{\infty}{\infty}$ & wrong
4 pts div. $1/x^2$ & correct
2 pts if & wrong
2 pts correct no work

48

* key *

4. Remember our function today is $f(x) = \dfrac{x^2}{x-1}$.

a) Make number lines for f, f', and f''. You MAY use the quotient rule.

$f(x) = \dfrac{x^2}{x-1}$

$f'(x) = \dfrac{2x(x-1) - x^2(1)}{(x-1)^2}$

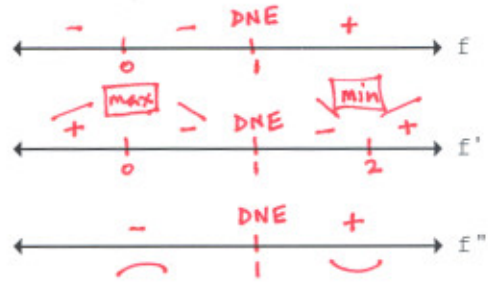
$= \dfrac{2x^2 - 2x - x^2}{(x-1)^2} = \dfrac{x^2-2x}{(x-1)^2}$

$= \dfrac{x(x-2)}{(x-1)^2} = \dfrac{x^2-2x}{x^2-2x+1}$

$f''(x) = \dfrac{(2x-2)(x^2-2x+1) - (x^2-2x)(2x-2)}{(x-1)^4}$

$= \dfrac{2x^3 - 4x^2 + 2x - 2x^2 + 4x - 2 - 2x^3 + 2x^2 + 4x^2 - 4x}{(x-1)^4}$

$= \dfrac{2x-2}{(x-1)^4} = \dfrac{2(x-1)}{(x-1)^4}$



5 each deriv (x2)

5 each # line (x3)

25 pts

b) List the (x, y)-values of any local extrema or inflection points of $f(x)$.

max at $(0, f(0)) = (0, \frac{0^2}{0-1}) = (0,0)$

min at $(2, f(2)) = (2, \frac{2^2}{2-1}) = (2,4)$ (no IPs)

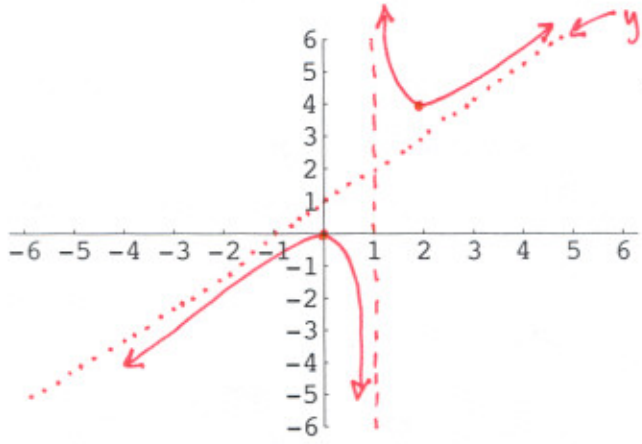
5 pts

2 pts (0,0)
2 pts (2,4)
1 pt no IP

-1 each y-val not given

-1 not saying if min/max

5. Make an extremely careful graph of $f(x)$ that includes *all* of the information on both sides of this exam, including the derivative information, an accurate graph of the slant asymptote, and the locations of any interesting points on the graph.



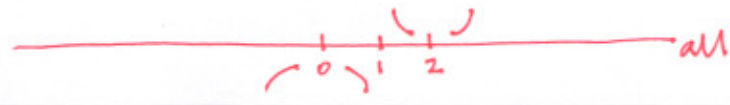
20 pts

5 { 2 +/-, 2 root, 2 V.A.

10 { 2 slant, 2 max, 2 min, 2 inc/dec, 2 concavity

4 overall

+2 free



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