231 TEST 1

You may use your notebook during the last half hour of this exam.
You may NOT use calculators, cell phones, loose papers, or peeking.

Math 231
February 3, 2011

Name: * key *
By printing my name I pledge to uphold the honor code.

12 × 3 pts

1. Fill in the blanks. You need not show work for these problems; do calculations on the scrap page, please.

if $x<1$ then $x<2$ — Write the contrapositive of the statement "If $x \geq 2$, then $x \geq 1$". #56, 0.4

if $x \geq 1$ then $x \geq 2$ — Write the converse of the statement "If $x \geq 2$, then $x \geq 1$". #56, 0.4

within $1/3$ — If we want to require that $3x+1$ is with distance 1 of 10, how close must x be to 3? T.F. 0.3

$f(a) > f(b)$ — If $f(x)$ is decreasing on $\mathbb{R}$ then $a<b$ implies what? #22, 0.5

one poss: $x^{1/2}$ (fractional power) — Give an example of a power function that is not a polynomial. #2c, 0.6

$f(x)=x$ (also $\frac{1}{x}$, others) — Give an example of a function $f(x)$ that is its own inverse. #2b, 0.7

$f(x)=0$ — Give an example of a function $f(x)$ that is both even and odd. #2c, 0.7

50 decibels — Suppose that a stereo speaker's optimal volume v, in decibels, is in the range described by the inequality $|v-85| \leq 35$. What is the lowest optimal volume for this speaker? technically oops! #66, 0.3

one poss: $a=-2$, $b=2$ (one neg one pos) — List two real numbers a and b for which $|a+b|$ is strictly less than $|a|+|b|$. #2c, 0.3

one pos: $a=-5$ (neg) — Give an example of a real number a with the property that $|a| = -a$, if possible. Caution on p. 6

$-1/3$ — Find the average rate of change of $f(x) = \frac{1}{x}$ on $[1,3]$. #77, 0.5

$[1,2) \cup (2,\infty)$ — Find the domain of the function $f(x) = \dfrac{\sqrt{x-1}}{x^2-4}$. #39, 0.5

36

like $f(-10) = \frac{-}{-} = +$
$f(1.5) = \frac{-}{+} = -$
$f(3) = \frac{-}{+} = -$
$f(10) = \frac{+}{+} = +$

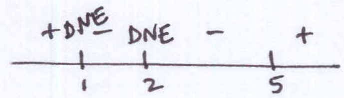
2. Calculate! Show your work clearly and put boxes around final answers.

a) Solve the inequality $\dfrac{x^2 - 7x + 10}{x^2 - 3x + 2} \leq 0$ by using a labeled number line.

10 pts

$\boxed{\#38, 0.3}$

$$\frac{\cancel{(x-2)}(x-5)}{\cancel{(x-2)}(x-1)} \leq 0$$



don't forget to put +/- here by plugging in values

$\boxed{[1, 2) \cup (2, 5]}$

b) Solve the equation $\dfrac{\frac{x+2}{x} - \frac{2}{x-1}}{x+1} = 0$. Be careful to omit any extraneous solutions.

10 pts

$\boxed{\#34, 0.2}$

$(x \neq -1)$ $\quad \dfrac{x+2}{x} - \dfrac{2}{x-1} = 0$

$(x+2)(x-1) - 2x = 0$
$x^2 + x - 2 - 2x = 0$
$x^2 - x - 2 = 0$
$(x+1)(x-2) = 0$

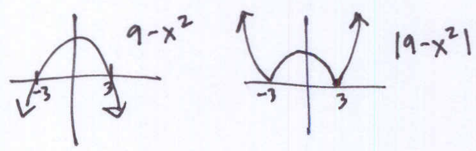
So $\cancel{x = -1}$ (extraneous), $x = 2$ is the only sol'n.

note you can check by plugging in

c) Write the function $f(x) = |9 - x^2|$ as a piecewise function that does not involve absolute values, where each piece is defined on an interval of x-values.

10 pts

$\boxed{\#37, 0.6}$



$9 - x^2$ is pos on $(-3, 3)$
and neg on $(-\infty, -3) \cup (3, \infty)$

$$\boxed{\text{so } |9 - x^2| = \begin{cases} -(9 - x^2), & \text{if } x < -3 \\ 9 - x^2, & \text{if } -3 \leq x \leq 3 \\ -(9 - x^2), & \text{if } x > -3 \end{cases}}$$

d) Use the values given in the table to fill in the missing values. There is only one way to correctly fill in the table. (You do not need to show work for this problem.)

$\boxed{\#19 \& 20, 0.7}$

10 pts

(-2 each wrong until points run out)

x	0	1	2
$f(x)$	3	4	1
$g(x)$	2	1	0
$(f \cdot g)(x)$	6	4	0
$(f \circ g)(x)$	1	4	3

40

8 x 3 pts

3. Give precise definitions, with mathematical notation, for each of the following. Be careful that you do not just list an associated property or give a rough description; actual definitions are called for here.

a) What is the definition of a ***rational number***?

a number that can be written in the form $\frac{p}{q}$, where p and q are integers

a key part

b) What is the definition of a ***function***?

f : A → B is a function if each element x of A is assigned to exactly one element f(x) of B.

c) What is the definition of a ***one-to-one function***?

$a \neq b \Rightarrow f(a) \neq f(b)$

· OR ·

$f(a) = f(b) \Rightarrow a = b$

for all a, b in dom(f)

a key part

d) What is the definition of a ***local maximum*** of a function?

a point $x = c$ such that for some $\delta > 0$ we have $f(c) \geq f(x)$ for all $x \in (c - \delta, c + \delta)$.

(notice = is okay)

e) What is the definition of a ***linear function***?

one that can be written in the form $f(x) = ax + b$, where a and b are real numbers

key parts

f) What is the definition of a ***rational function***?

can be written in the form $f(x) = \frac{p(x)}{q(x)}$, where p(x) and q(x) are polynomials

g) What is the definition of an ***odd function***?

$f(-x) = -f(x)$, for all x in dom(f)

a key part

h) Draw a picture of two of your favorite animals having a battle for three free points.

← mystery animals!

Survey for 2 bonus points: How do you think you did? What is a question or topic that could have been on this exam, but wasn't?

true/false
proof of quadratic formula
"Land of Gore" inequalities
midpoint & distance formulas
showing a fncn is even/odd

transformations of graphs
listing graphical properties
triangle inequality
proofs
negating quantifiers & and/or
unions and intersections

graph piecewise

24