231 EXAM 1

You may use your notebook during the last 15 minutes of this exam.

You may NOT use calculators, cell phones, loose papers, or peeking.

Math 231
September 17, 2013

Name: * Key *

By printing my name I pledge to uphold the honor code.

1. Determine if each of the following statements are true or false. For those that are true, *briefly* describe why they are true. For those that are false, provide an explicit counterexample.

a) For all real numbers x, there exists some y such that $x < y$. TRUE

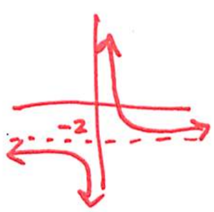
every real number has at least one larger real number.

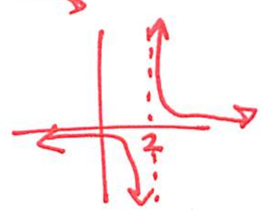
b) For all real numbers x, there exists some y such that $x = y^2$. FALSE

Counterexample: if $x = -1$ then there is no real y w/ $-1 = y^2$

(give pts if they say true and mention cx #

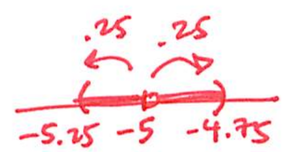
2. Sketch labeled graphs of $f(x) = \dfrac{1}{x} - 2$ and $g(x) = \dfrac{1}{x-2}$ (and say which is which!).





So $x \neq -5$

3. The solution to the inequality $0 < |x + 5| < 0.25$ is:



A) $(-5.25, -5) \cup (-5, -4.75)$

B) $(-5.25, 0) \cup (0, 5.25)$

C) $(-5.25, -4.75)$

D) $(-5.25, 4.75)$

4. The inverse of the function $f(x) = 3 - 4x$ is:

mult. by -4 & add 3
sub. 3 and div. by -4

$y = 3 - 4x$
$x = 3 - 4y$ (swap)
$x - 3 = -4y$
$y = \dfrac{x-3}{-4}$

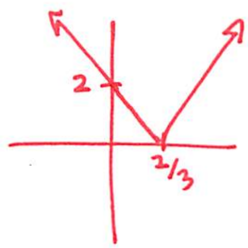
A) $f^{-1}(x) = \dfrac{1}{3+4x}$

B) $f^{-1}(x) = \dfrac{1}{3-4x}$

C) $f^{-1}(x) = \dfrac{x-3}{-4}$

D) $f^{-1}(x) = \dfrac{x}{-4} - 3$

5. Sketch a labeled graph of the function $f(x) = |2 - 3x|$, and then express $f(x)$ algebraically as a piecewise-defined function that does not involve absolute values.



$2 - 3x = 0$

when

$-3x = -2$

$x = 2/3$

$+ \quad\quad -$

$2/3$

$$|2-3x| = \begin{cases} 2-3x, & \text{if } x \le 2/3 \\ -(2-3x), & \text{if } x > 2/3 \end{cases}$$

6. Find the domain of the function $f(x) = \dfrac{\sqrt{x^2 - 4}}{x^2 - 9}$. Show your work and put your final answer in interval notation with a box around it.

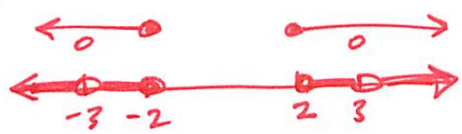
need $x^2 - 4 \ge 0$

and $x^2 - 9 \ne 0$

$$\Rightarrow \begin{cases} x^2 \ge 4 \\ x^2 \ne 9 \end{cases} \Rightarrow \begin{cases} x \in (-\infty, -2] \cup [2, \infty) \\ x \ne \pm 3 \end{cases}$$

$(x+2)(x-2)$

$+ \quad -2 \quad - \quad 2 \quad +$



$$\boxed{(-\infty, -3) \cup (-3, -2] \cup [2, 3) \cup (3, \infty)}$$

7. Prove that the sum of two odd integers is always even. Make sure that your argument is clear and uses mathematical notation and definitions.

<u>proof.</u> Suppose a and b are odd.

then $a = 2k + 1$, $b = 2l + 1$ for some k and l integers.

thus $a + b = (2k+1) + (2l+1) = 2\underbrace{(k+l+1)}_{\text{an integer}}$,

So $a+b$ is even. $\square$