231 EXAM 2

You may use your notebook during the last 15 minutes of this exam.

You may NOT use calculators, cell phones, loose papers, or peeking.

Math 231
October 15, 2013

Name: * Key *

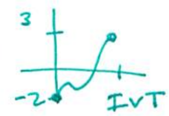
By printing my name I pledge to uphold the honor code.

1. Determine whether each of the following is True (T) or False (F).

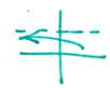
2 each
12pts

T **F** Every algebraic function is continuous on its entire domain.

T **F** If f is continuous everywhere, and if $f(0) = -2$ and $f(4) = 3$, then $f(x)$ must have a root somewhere in $(0, 4)$.



T **F** If $\lim_{x \to -\infty} f(x) = 3$, then the graph of f has a horizontal asymptote at $y = 3$.



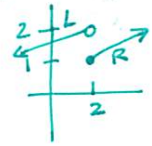
T **F** If f is continuous at $x = c$, then f is differentiable at $x = c$.

T **F** If $f(c) = 10$, then $\lim_{x \to c} f(x) = 10$.

T **F** Saying that $0 < |x - c|$ is the same thing as saying that $x \neq c$.

since $0 \leq |x-c|$ always, so only excluding $0 = |x-c|$

2. If $\lim_{x \to 2^-} f(x) = 2$, $\lim_{x \to 2^+} f(x) = 1$, and $f(2) = 1$, then f is:



A) continuous at $x = 2$

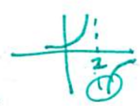
B) left but not right continuous at $x = 2$

C) right but not left continuous at $x = 2$

D) neither left or right continuous at $x = 2$

3 each
9 pts

3. Find $\lim_{x \to 2^+} \frac{1}{2 - x}$. $\to \frac{1}{2-2^+} \to \frac{1}{0^-} \to -\infty$



A) 0 **B)** ∞ **C)** $-\infty$ **D)** indeterminate

4. Find $\lim_{x \to \infty} \frac{1}{2 - x}$. $\to \frac{1}{2-\infty} \to \frac{1}{-\infty} \to 0$

A) 0 **B)** ∞ **C)** $-\infty$ **D)** indeterminate

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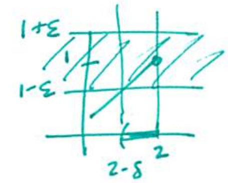
2 2
2 2
2
2

12 pts

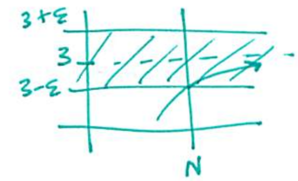
5. Fill in the blanks with interval notation:
If $\lim_{x \to 2^-} f(x) = 1$, then for all $\epsilon > 0$, there is some $\delta > 0$ such that

if $x \in$ $(2-\delta, 2)$, then $f(x) \in$ $(1-\varepsilon, 1+\varepsilon)$.



6. Fill in the blanks with interval notation:
If $\lim_{x \to \infty} f(x) = 3$, then for all $\epsilon > 0$, there is some $N > 0$ such that

if $x \in$ (N, ∞), then $f(x) \in$ $(3-\varepsilon, 3+\varepsilon)$.



7. Fill in the blanks:

If $|f(x) - L| < \epsilon$, then $L-\varepsilon$ $< f(x) <$ $L+\varepsilon$.

8. Fill in the blank with interval notation:

If $0 < |x-2| < 0.5$, then $x \in$ $(1.5, 2) \cup (2, 2.5)$.

9. Calculate the limit $\lim_{x \to \infty} (\sqrt{x} - x)$. Show all work.

8 pts

$\lim_{x \to \infty} (\sqrt{x} - x) \to \sqrt{\infty} - \infty \to \infty - \infty$ indeterminate (don't know yet)

$= \lim_{x \to \infty} \sqrt{x}(1-\sqrt{x}) \to \infty(1-\infty) \to \infty(-\infty) \to \boxed{-\infty}$

10. Use either the $h \to 0$ or $z \to x$ definition of derivative to show that $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$.

9 pts

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\left(\frac{x-(x+h)}{(x+h)x}\right)}{h}$

$= \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - h}{h(x+h)x} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h)x} = \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{x \cdot x} = -\frac{1}{x^2}$.

OR

$\lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{z \to x} \frac{\frac{1}{z} - \frac{1}{x}}{z - x} = \lim_{z \to x} \frac{\left(\frac{x-z}{zx}\right)}{z - x}$

$= \lim_{x \to z} \frac{\overset{-1}{\cancel{x-z}}}{\cancel{(z-x)}zx} = \lim_{x \to z} \frac{-1}{zx} = \frac{-1}{x \cdot x} = \frac{-1}{x^2}$.

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