

231 Quiz 3

Name: _____

October 30, 2013

Name: * key *

Name: _____

Work in groups. You must do all problems together, discussing and agreeing on each answer; do not split up the work. Hand in one copy of the quiz with a very clear, concise writeup. You may use your Notebooks.

1. Use the DEFINITION OF DERIVATIVE to prove the following "reciprocal rule" for differentiation: (Like #90 in 2.3)

$$\frac{d}{dx} \left(\frac{1}{f(x)} \right) = \frac{-f'(x)}{(f(x))^2}$$

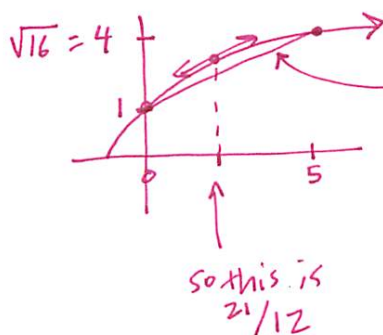
Do NOT use the quotient rule or the chain rule; this is a proof using the definition of derivative.

8 pts

$$\begin{aligned} \frac{d}{dx} \left(\frac{1}{f(x)} \right) &= \lim_{h \rightarrow 0} \frac{\frac{1}{f(x+h)} - \frac{1}{f(x)}}{h} = \lim_{h \rightarrow 0} \frac{\frac{f(x) - f(x+h)}{f(x+h)f(x)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x) - f(x+h)}{h f(x+h)f(x)} = - \left(\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \right) \left(\lim_{h \rightarrow 0} \frac{1}{f(x+h)f(x)} \right) \\ &= -f'(x) \left(\frac{1}{(f(x))^2} \right) = \frac{-f'(x)}{(f(x))^2} \quad \square \end{aligned}$$

2. Use derivatives and algebra to find the exact values of all $c \in [0, 5]$ for which the function $f(x) = \sqrt{3x+1}$ satisfies the Mean Value Theorem on the interval $[0, 5]$. Show your work clearly for full credit. A picture helps. (Like #52 in 3.1)

8 pts



AROC from $(0, 1)$ to $(5, 4)$ = $\frac{4-1}{5-0} = \frac{3}{5}$

$$f'(x) = \frac{1}{2}(3x+1)^{-1/2} (3) = \frac{3}{5}$$

$$\frac{1}{\sqrt{3x+1}} = \frac{2}{5}$$

$$\frac{5}{2} = \sqrt{3x+1}$$

$$\left(\frac{5}{2}\right)^2 = 3x+1$$

$$x = \frac{\frac{25}{4} - 1}{3} = \frac{25}{12} - \frac{1}{3} = \frac{21}{12} = \frac{7}{4}$$