Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).

- Make sure that you follow the directions in each problem and that your answer matches what is asked for.

- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!

- Good luck!

Some volume and surface areas you may or may not need:
A sphere with radius $r$ has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.
A cylinder with radius $r$ and height $h$ has volume $\pi r^2h$, "lateral" or "side" surface area $2\pi rh$, and top and bottom surface area each $\pi r^2$. 
1. (20 pts) Answer true or false, fill in the blanks, or give short answers to each of the following questions.

   a. (2 pts)  The product of two algebraic functions is itself an algebraic function.

   b. (2 pts)  \( f(x) = 2e^x + 1 \) has a constant doubling time.

   c. (2 pts)  \( f(x) = 500x^{1500} \) dominates \( g(x) = 0.001(1.003)^x \) as \( x \to \infty \).

   d. (2 pts)  Use interval notation to fill in the blank: If \( b \in \underline{\hspace{2cm}} \), then \( \lim_{x \to \infty} b^x = 0 \).

   e. (3 pts)  \( f(x) = \log_b x \) has domain \underline{\hspace{2cm}} and range \underline{\hspace{2cm}}.

   f. (3 pts)  Give a mathematical definition of the number \( e \).

   g. (3 pts)  If \( f(x) \) has a vertical cusp at \( x = c \), what can you say about \( \lim_{x \to c^-} f'(x) \) and \( \lim_{x \to c^+} f'(x) \)?

   h. (3 pts)  If the volume \( V \) and radius \( r \) of a sphere are functions of time, find an equation that relates the rate of change of the volume and the rate of change of the radius.
2. (18 pts) Show all work clearly and in order.
   
a. (6 pts) Find the derivative of \( f(x) = \ln(e^{2x} + 4) \).
   
b. (6 pts) Given that \( \sqrt{y} + 1 = 5xy \), find \( \frac{dy}{dx} \).
   
c. (6 pts) Given that \( f(2) = 3 \), \( f'(2) = 5 \), \( g(2) = 2 \), and \( g'(2) = 4 \), find \( \frac{d}{dx} \bigg|_{x=2} (g(x) \ln(f(x))) \).

3. (12 pts) Show all work in each part below. No calculators.
   
a. (6 pts) Solve the equation \( \frac{1}{4^x} - \frac{5}{2^x} + 4 = 0 \).
   
b. (6 pts) Find the domain of the function \( f(x) = \frac{\sqrt{5-x}}{\ln(x-2)} \).
4. (8 pts) Prove that for any \( k < 0 \), the function \( f(x) = e^{kx} \) is always decreasing and concave up.

5. (8 pts) Use the fact that \( \ln x \) is the inverse of \( e^x \) to prove that \( \frac{d}{dx}(\ln x) = \frac{1}{x} \).

(Hint: The definition of inverse functions will give you an equation that you can differentiate.)

6. (8 pts) The cost of the material for the top and bottom of a cylindrical can is 5 cents per square inch. The material for the “side” of the can (the curvy part) costs only 2 cents per square inch. If the can must hold 400 cubic inches of liquid, what radius will produce the cheapest can?
7. (8 pts) Suppose a quantity $Q(t)$ doubles every seven years. What is its tripling time? Show your work clearly and in order.

8. (18 pts) Calculate each of the following limits by hand. Show all work clearly.

   a. (6 pts) $\lim_{x \to 0^+} \frac{x}{\log_2 x}$

   b. (6 pts) $\lim_{x \to \infty} x^2 3^{-x}$

   c. (6 pts) $\lim_{x \to \infty} \frac{2}{3 + e^{-25x}}$
Survey Questions: (2 extra credit points)

Name a question or topic that could have been on this test, but wasn’t.

How do you think you did?

SPACE FOR SCRAP WORK