# TEST I

Math 232
September 26, 2002

**Name:** _____

By writing my name I swear by the honor code.

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

**Some volume and surface areas you may or may not need:**
A sphere with radius $r$ has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.
A cylinder with radius $r$ and height $h$ has volume $\pi r^2 h$, "lateral" or "side" surface area $2\pi rh$, and top and bottom surface area each $\pi r^2$.

**1.** (*20 pts*) Answer true or false, fill in the blanks, or give short answers to each of the following questions.

**a.** (*2 pts*) **T** **F** The product of two algebraic functions is itself an algebraic function.

**b.** (*2 pts*) **T** **F** $f(x) = 2e^x + 1$ has a constant doubling time.

**c.** (*2 pts*) **T** **F** $f(x) = 500x^{1500}$ dominates $g(x) = 0.001(1.003)^x$ as $x \to \infty$.

**d.** (*2 pts*) Use interval notation to fill in the blank: If $b \in$ _____ , then $\lim_{x\to\infty} b^x = 0$.

**e.** (*3 pts*) $f(x) = \log_b x$ has domain _____ and range _____ .

**f.** (*3 pts*) Give a mathematical definition of the number $e$.

**g.** (*3 pts*) If $f(x)$ has a vertical cusp at $x = c$, what can you say about $\lim_{x\to c^-} f'(x)$ and $\lim_{x\to c^+} f'(x)$?

**h.** (*3 pts*) If the volume $V$ and radius $r$ of a sphere are functions of time, find an equation that relates the rate of change of the volume and the rate of change of the radius.

**2.** (*18 pts*) Show all work clearly and in order.

**a.** (*6 pts*) Find the derivative of $f(x) = \ln(e^{2x} + 4)$.

**b.** (*6 pts*) Given that $\sqrt{y} + 1 = 5xy$, find $\frac{dy}{dx}$.

**c.** (*6 pts*) Given that $f(2) = 3$, $f'(2) = 5$, $g(2) = 2$, and $g'(2) = 4$, find $\left.\frac{d}{dx}\right|_{x=2} (\, g(x)\ln(f(x))\,)$.

**3.** (*12 pts*) Show all work in each part below. No calculators.

**a.** (*6 pts*) Solve the equation $\frac{1}{4^x} - \frac{5}{2^x} + 4 = 0$.

**b.** (*6 pts*) Find the domain of the function $f(x) = \frac{\sqrt{5-x}}{\ln(x-2)}$.

**4.** (*8 pts*) Prove that for any $k < 0$, the function $f(x) = e^{kx}$ is always decreasing and concave up.

**5.** (*8 pts*) Use the fact that $\ln x$ is the inverse of $e^x$ to prove that $\frac{d}{dx}(\ln x) = \frac{1}{x}$.
*(Hint: The definition of inverse functions will give you an equation that you can differentiate.)*

**6.** (*8 pts*) The cost of the material for the top and bottom of a cylindrical can is 5 cents per square inch. The material for the "side" of the can (the curvy part) costs only 2 cents per square inch. If the can must hold 400 cubic inches of liquid, what radius will produce the cheapest can?

**7.** (*8 pts*) Suppose a quantity $Q(t)$ doubles every seven years. What is its tripling time? Show your work clearly and in order.

**8.** (*18 pts*) Calculate each of the following limits by hand. Show all work clearly.

**a.** (*6 pts*) $\lim_{x \to 0^+} \dfrac{x}{\log_2 x}$

**b.** (*6 pts*) $\lim_{x \to \infty} x^2 3^{-x}$

**c.** (*6 pts*) $\lim_{x \to \infty} \dfrac{2}{3 + e^{.25x}}$

**Survey Questions:** (2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

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**SPACE FOR SCRAP WORK**