TEST II

Math 232
October 24, 2002

Name: _____
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

Some formulas you may or may not need:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

1. (*14 pts*) Determine whether each statement is true (T) or false (F).

a. (*2 pts*) **T** **F** We used the limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to show that $\sin x$ is continuous at $x = 0$.

b. (*2 pts*) **T** **F** We used the limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to show that $\frac{d}{dx}(\sin x) = \cos x$.

c. (*2 pts*) **T** **F** $\sec^{-1} x = \cos x$.

d. (*2 pts*) **T** **F** $\sec^{-1} x = \dfrac{1}{\cos^{-1} x}$.

e. (*2 pts*) **T** **F** $\sec^{-1} x = \dfrac{1}{|x|\sqrt{x^2-1}}$.

f. (*2 pts*) **T** **F** The restricted domain of $\cos x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

g. (*2 pts*) **T** **F** The function $\tan^{-1} x$ is defined for all real numbers.

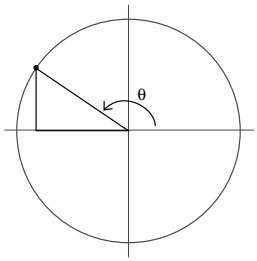
2. (*15 pts*) Fill in the blanks or circle answers, as appropriate.

a. (*3 pts*) Suppose θ is the angle shown below. Circle ***all*** of the following that are true. (Note: Do *not* attempt to guess the value of the angle θ; just use the picture.)

$\sin\theta < \cos\theta$

$-1 < \tan\theta < 1$

$\cos^{-1}(-\frac{1}{2}) < \theta$



b. (*3 pts*) If θ is the angle shown above in part (a), which of the following angle measures is the closest in value to θ? (Circle ***one*** of the options.)

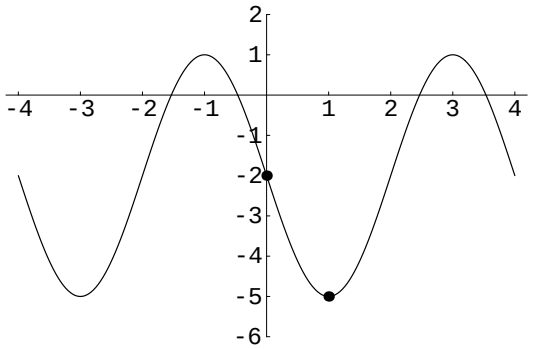
1 radian 2 radians 30 radians 40° -40°

c. (*3 pts*) $\dfrac{d}{dx}(\sec^2(\ln x)) =$ _____.

d. (*3 pts*) $\dfrac{d}{dx}(\sin^{-1}(x^2)) =$ _____.

e. (*3 pts*) If the ***derivative*** is $f'(x) = \dfrac{3}{1+4x^2}$, then $f(x) =$ _____.

3. (*7 pts*) Find a general cosine function $f(x) = A\cos(B(x + C)) + D$ that has the graph shown below. Note that the graph passes through the points $(0, -2)$ and $(1, -5)$. Circle your final answer.



4. (*14 pts*) The following two questions involve the function $(\sin x)^x$. Do all work by hand and show your work clearly. Circle your final answers.

a. (*7 pts*) Find $\lim_{x \to 0^+} (\sin x)^x$.

b. (*7 pts*) Find $\dfrac{d}{dx}((\sin x)^x)$.

5. (*14 pts*) For each part below, do all work by hand (no calculators). Your work should include an angle sketched on the unit circle and a triangle with labeled sides. Circle your final answers.

a. (*7 pts*) Find the coordinates (x, y) of the point where the terminal edge of the angle $-\frac{7\pi}{6}$ meets the unit circle.

b. (*7 pts*) If $\theta = \sin^{-1}(-\frac{1}{3})$, find the *exact* value of $\cos\theta$.

6. (*15 pts*) Fill in each of the blanks below with a number. No calculators except for simple arithmetic (adding up or multiplying numbers, etc.).

a. (*3 pts*) $\displaystyle\sum_{k=4}^{9} \frac{1}{k-1} =$ _____.

b. (*3 pts*) $\displaystyle\sum_{k=3}^{500} (3k^2 - 2) =$ _____.

c. (*3 pts*) If $\lim_{x\to 0} \ln(f(x)) = -2$, then $\lim_{x\to 0} f(x) =$ _____ .

d. (*3 pts*) If $\sin\theta = \frac{2\sqrt{6}}{5}$ and $\cos\theta = \frac{1}{5}$, then $\cos 2\theta =$ _____ .

e. (*3 pts*) Write anything you like in this box:

7. (*7 pts*) Find $\lim_{x \to 1} \dfrac{(x-1)^3}{\sin^2(x-1)}$ without using L'Hôpital's Rule. Show your work clearly and circle your final answer.

8. (*14 pts*) In each proof below make sure that your argument is clear and presented in a logical order. Justify any relevant steps.

a. (*7 pts*) Prove algebraically that the function $f(x) = \dfrac{\cos x}{\sin x + \cot x}$ is odd.

b. (*7 pts*) Prove that $\sec^2(\tan^{-1} x) = 1 + x^2$ for all values of x.

Survey Questions: (2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK