

TEST III

Math 232
November 21, 2002

Name: _____
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has six problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (8 pts) Determine whether each of the following statements is true (T) or false (F).

a. (2 pts) T F The function $A(x) = \int_0^x (3 - t) dt$ is increasing on $[0, 3]$.

b. (2 pts) T F If $f'(x) = F(x)$ then $\int_a^b f(x) dx = [F(x)]_a^b$.

c. (2 pts) T F If f is negative on $[-3, 2]$, then $-\int_{-3}^2 f(x) dx$ is negative.

d. (2 pts) T F $\int \frac{1}{1 + \sqrt{x}} dx = 2\sqrt{x} - 2 \ln |1 + \sqrt{x}| + C$.

2. (20 pts) Fill in the blanks and circle answers, as appropriate.

a. (3 pts) If $f(4) = 6$ and $\int_{-1}^4 f'(x) dx = 8$, then $f(-1) = \underline{\hspace{2cm}}$.

b. (3 pts) The function $A(x) = \int_3^{x^2} \sin t dt$ is a composition of functions $A(x) = g(h(x))$ with:

$g(x) = \underline{\hspace{2cm}}$ and $h(x) = \underline{\hspace{2cm}}$.

c. (3 pts) To solve the integral $\int \frac{1}{(x - 5)^2 + 9}$ you would start by using the trigonometric substitution:

$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

d. (3 pts) Write down *two* integrals that will have the form $\int e^u du$ after a substitution of variables.

$\underline{\hspace{2cm}} \quad \underline{\hspace{2cm}}$

e. (7 pts) The Riemann sum below approximates the area under the graph of some function $f(x)$ from $x = a$ to $x = b$. Determine the type of approximation and identify $f(x)$, a , b , N , Δx , and x_k .

Circle one: (LHS) (RHS) (Midpoint) (Trapezoid)

$$\sum_{k=1}^4 \left(1 + \frac{k}{2}\right)^2 \left(\frac{1}{2}\right) \quad N = \underline{\hspace{1cm}}, \quad \Delta x = \underline{\hspace{1cm}}, \quad x_k = \underline{\hspace{2cm}}.$$

$$f(x) = \underline{\hspace{2cm}}, \quad a = \underline{\hspace{1cm}}, \quad b = \underline{\hspace{1cm}}.$$

f. (1 pt) Write any word in this blank for one point: $\underline{\hspace{2cm}}$.

3. (24 pts) Solve each of the following integrals. Show all work clearly and in order.

a. (8 pts) $\int \tan^5 x \sec^3 x \, dx$

b. (8 pts) $\int \frac{4+x^2}{x} \, dx$

c. (8 pts) $\int \frac{x^2}{(1-x^2)^{\frac{3}{2}}} \, dx$

4. (8 pts) Find the *exact* average value of the function $f(x) = \frac{1}{x^2}$ on the interval $[2, 5]$.

5. (16 pts) This problem concerns the definite integral $\int_1^3 x e^x dx$.

a. (8 pts) Approximate the definite integral above using the Midpoint Sum approximation with $N = 4$ rectangles. (Note: You do *not* have to use any sigma notation to do this problem!)

b. (8 pts) Find the *exact* value of the definite integral above (by using the Fundamental Theorem of Calculus and integration techniques).

6. (24 pts) Solve each of the following integrals. Show all work clearly and in order.

a. (8 pts) $\int \tan^3 x \, dx$

b. (8 pts) $\int \sin^{-1} x \, dx$

c. (8 pts) $\int \sin^4 x \, dx$

Survey Questions: (2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK