Homework for Week 1 **Math 232 Spring 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 7.1 3, 5, 11, 13, 15, 16, 18, 26, 27, 28, 38, 39, 40, 41, 42, 43, 51.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test.

Section 7.3

3. Combine the graphs of $y = \sqrt{\frac{1}{3}x^2 - \frac{16}{3}}$ and $y = -\sqrt{\frac{1}{3}x^2 - \frac{16}{3}}$.
5. Since $(0, 1)$ is a solution to the equation, that point must be on the graph; only Figure 8 has this point.
11. $s^2 + 2rss'$.
15. This is an explicit function: $y = \sqrt[5]{2x - 3}$.
16. This is an implicit function, since (for example) $x = 0$ produces two outputs, $y = 0$ and $y = \sqrt[3]{3}$. It is not possible to solve for y.
26. $\frac{1}{2}(3y-1)^{-\frac{1}{2}}(3y') = 5y + 5xy'$, so $y' = \dfrac{5y}{\frac{3}{2}(3y-1)^{-\frac{1}{2}} - 5x}$.
27. $\dfrac{2yy'(3y-1) - (y^2+1)(3y')}{(3y-1)^2} = 1$, so $y' = \dfrac{(3y-1)^2}{2y(3y-1) - 3(y^2+1)}$.
38. When $x = 1$ we have $y^3 + y + 2 = 0$. Using synthetic division we can write this as $(y+1)(y^2 - y + 2) = 0$. The quadratic is irreducible, so the only solution is $y = -1$. Therefore $(1, -1)$ is the only point on the graph with an x-coordinate of 1.
39. $3y^2y' + y + xy' = 0$, so $y' = \dfrac{-y}{3y^2 + x}$. Thus $\frac{dy}{dx}\big|_{x=1,\, y=-1} = \frac{-(-1)}{3(-1)^2+1} = \frac{1}{4}$.
40. When $y = 1$ we have $1 + x + 2 = 0$, and thus $x = -3$; thus $(-3, 1)$ is the only point on the graph with a y-coordinate of 1.
41. By problem 39, $\frac{dy}{dx}\big|_{x=-3,\, y=1} = \frac{-1}{3(1)^2+(-3)} = \frac{-1}{0}$. Thus the slope of the tangent line at $(-3, 1)$ is "infinite", *i.e.* the tangent line is vertical at that point.
42. $y' = \frac{-y}{3y^2+x}$ is zero if $y = 0$ (but $3y^2 + x \neq 0$). When $y = 0$ the original equation is $0 + 0 + 2 = 0$, which has no solutions. Therefore there aren't any points on the graph with a y-coordinate of 0, and thus there aren't any points on the graph with a horizontal tangent line.
43. The tangent line will be vertical when the reciprocal of the derivative is zero, *i.e.* when $3y^2 + x = 0$ (but $y \neq 0$). Plug $x = -3y^2$ into the original equation and solve to find $x = -3$, $y = 1$ as the only solution. Thus the graph has a vertical tangent line only at the point $(-3, 1)$ (also see problem 41).

The graph of $y^3 + xy + 2 = 0$ (problems 38–43) looks like: