Homework for Week 11 **Math 232 Spring 2002**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 12.4 1, 5, 9, 11, 13, 14, 17, 19, 21, 33, 34, 37, 39, 43, 46, 48, 51, 55, 57, 59, 60, 61.

Section 13.1 1, 3, 4, 6, 7, 9, 10, 15, 16, 18, 19, 22, 23, 26, 29, 32, 36, 37, 38, 41, 44, 47, 50, 51, 52, 55, 58, 59, 61, 62, 65, 69, 76.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 12.4

9. Hint: See Figure 8.
13. The average height of the function should be ten, while the slope from $(-2, f(-2))$ to $(5, f(5))$ should be -3.
14. See page 48. What does this have to do with the way that we define the average value of a function?
17. One approximation is an average value of ≈ 3.
19. Part (a): $\int_{-3}^{7} f(x)\,dx$.

    Part (b): $-\int_{-3}^{-2} f(x)\,dx + \int_{-2}^{1} f(x)\,dx - \int_{1}^{6} f(x)\,dx + \int_{6}^{7} f(x)\,dx$.
21. $\int_{-2}^{1} (f(x) - g(x))\,dx + \int_{1}^{3} (g(x) - f(x))\,dx$.
33. False.
34. Using a Left Hand Sum with $N = 8$ we have **(a)** -11.1973 and **(b)** 12.2229. (To get the true area, consider the area of each rectangle to be positive, regardless of whether it is above or below the x-axis.)
39. $(f(-1) + f(-.75) + f(-.5) + f(-.25) + f(0) + f(.25) + f(.5) + f(.75))(\frac{1}{8}) \approx 0.990978$.
46. Part (a): $\int_{-1}^{3} (3x^2 + 5x - 2)\,dx = 40$ (use integration formulas).

    Part (b): $-\int_{-1}^{\frac{1}{3}} (3x^2 + 5x - 2)\,dx + \int_{\frac{1}{3}}^{3} (3x^2 + 5x - 2)\,dx \approx 3.85 + 43.85 = 47.7$.
51. $\int_{-3}^{-1} (x^2 - (x + 2))\,dx + \int_{-1}^{2} ((x + 2) - x^2)\,dx + \int_{2}^{3} (x^2 - (x + 2))\,dx = \frac{26}{3} + \frac{9}{2} + \frac{11}{6} = 15$.
57. $\frac{1}{3 - 0}\int_{0}^{3} (x^2 - 2x - 1)\,dx = \frac{1}{3}(-3) = -1$.

Section 12.4 (continued)

59. Part (a): You'll have to solve a few equations to determine the limits of integration. For example, $f(x) = 55$ when $0.5(x-12)^2 = 55$, *i.e.* when $x = 12 \pm \sqrt{\frac{55}{0.5}}$, or approximately $x \approx 1.51$ and $x \approx 22.49$ (we only care about the first value; why?). The area of the "W" is: $\int_{1.51}^{5.29} (55 - f(x))\, dx + \int_{5.29}^{18} (r(x) - f(x))\, dx + \int_{18}^{30.71} (s(x) - g(x))\, dx + \int_{30.71}^{34.49} (55 - g(x))\, dx.$
Part (b): The area is approximately 561.9 square feet (too big!).

60. Part (a): $(f(1) + f(2) + f(3) + f(4) + f(5))(\frac{1}{5}) \approx 3.75$ feet.
Part (b):
$(f(.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4) + f(4.5) + f(5))(\frac{1}{10}) \approx 3.4$ feet.
Part (c): $\dfrac{1}{5-0} \displaystyle\int_0^5 (.25t^2 + 1)\, dt$
Part (d): $(\frac{1}{5})(.25(\frac{1}{3}(5^3 - 0^3)) + 1(5-0)) \approx 3.08$ feet.

61. Part (a): Solve $s(t) = 0$ to find $t = \frac{5\sqrt{2}}{3} \approx 2.357$ seconds.
Part (b): Average distance is $\dfrac{1}{2.357 - 0} \displaystyle\int_0^{2.357} (50 - 9t^2)\, dt \approx 33.3337$ feet.
Part (c): Average velocity is $\frac{s(2.357) - s(0)}{2.357 - 0} = \frac{0 - 50}{2.357} \approx -21.213$ feet per second.
Part (d): $v(t) = -18t$, so average velocity is $\frac{1}{2.357-0} \int_0^{2.357} (-18t)\, dt \approx -21.213$ feet per second.
Part (e): The area under the velocity graph is the total change in distance! (In this case, 50 feet.) In general, $\int_a^b v(t)\, dt = s(b) - s(a)$; think about this - it will be a big part of the next chapter!

Section 13.1

3. (a) $\displaystyle\int x^6\, dx = \frac{1}{7}x^7 + C$; (b)$\frac{1}{7}x^7$ is an antiderivative of x^6; (c) the derivative of $\frac{1}{7}x^7$ is x^6, *i.e.* $\frac{d}{dx}(\frac{1}{7}x^7) = x^6$.
9. True.
10. False.
15. False.
16. False.
19. $\frac{d}{dx}(x(\ln x - 1)) = 1(\ln x - 1) + x(\frac{1}{x} - 0) = \ln x - 1 + 1 = \ln x$.
22. One easy example is $f(x) = x$, $g(x) = x^2$.
26. $f'(x) = 2x^{-\frac{1}{3}}$ implies that $f(x) = 2(\frac{2}{3}x^{\frac{2}{3}} + C = \frac{4}{3}x^{\frac{2}{3}} + C$. Then $f(1) = 2$ implies that $2 = \frac{4}{3}(1)^{\frac{2}{3}} + C$, so $x = 2 - \frac{4}{3} = \frac{2}{3}$. Therefore $f(x) = \frac{4}{3}x^{\frac{2}{3}} + \frac{2}{3}$.
27. $f(x) = \frac{5}{3}\ln|3x - 2| + 8$.
32. Hint: Multiply out the integrand.
36. Hint: $\frac{x+1}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$.
37. $\frac{3.2}{\ln(1.43)}(1.43)^x - 50x + C$.
38. $\frac{4}{2}e^{2x-3} + C$.
41. $3\sec x + C$.
44. $\frac{7}{2}\sin^{-1}(2x) + C$.

Section 13.1 (continued)

47. $\frac{a}{b}e^{bx+c} + C.$

50. $\frac{1}{ab}\tan(ax) + C.$

51. $\frac{a}{b}\tan^{-1}(bx) + C.$

52. $x^2e^x + C.$

55. $\ln|3x^2 + 1| + C.$

58. $\frac{1}{18}(x^3 + 1)^6 + C.$

59. $\frac{x^2}{\ln x} + C.$

61. Part (a): $\frac{d}{dx}(F(x)) = \frac{d}{dx}(G(x) + C) \quad \Rightarrow \quad \frac{dF}{dx} = \frac{dG}{dx} + 0 \quad \Rightarrow \quad \frac{dF}{dx} = \frac{dG}{dx}.$
Part (b):
$F'(x) = G'(x) \quad \Rightarrow$ F'(x)-G'(x)=0 $\Rightarrow \quad \frac{d}{dx}(F(x) - G(x)) = 0 \quad \Rightarrow \quad F(x) - G(x)$ is a constant (say C).
Part (c): *All* antiderivatives of a function differ by a constant from each other, so writing $\int f(x)\, dx = F(x) + C$ describes *all* the antiderivative of $f(x)$, and makes sense for *any* antiderivative $F(x)$.

62. $\frac{d}{dx}(\frac{1}{k+1}x^{k+1}) = \frac{1}{k+1}(k + 1)x^{(k+1)-1} = x^k.$

65. $\frac{d}{dx}(\frac{1}{k}e^{kx}) = \frac{1}{k}(ke^{kx}) = e^{kx}.$

69. $\frac{d}{dx}(-\cot x) = -(-\csc^2 x) = \csc^2 x.$

76. See the reading.