Homework for Week 15 **Math 232 Spring 2002**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 15.3 1, 3, 4, 5, 6, 8, 10, 13, 14, 16, 19, 24*, 27, 31, 35, 40, 41, 42, 45*, 46, 47, 48, 52, 53, 55.

Section 15.4 1, 2, 8, 9, 12, 13*, 14, 15, 16, 17, 19, 23*, 26, 28, 30*, 33.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 15.3

1. Part (a): $\pi(1.5)^2(3) - \pi(1)^2(3) \approx 11.781$.
   Part (b): $2\pi(1.25)(3)(0.5) \approx 11.781$.
   Part (c): $2\pi(2.25)(3)(0.5) \approx 21.2058$.
6. Part (a): $\Delta x = 0.5$, $x_0 = 0, x_1 = 0.5, x_2 = 1, x_3 = 1.5, x_4 = 2$.
   Part (b): $x_1^* = 0.25, x_2^* = 0.75, x_3^* = 1.25, x_4^* = 1.75$.
   Part (c): $f(x_1^*) = (.25)^2 = 0.0625, f(x_2^*) = (0.75)^2 = 0.5625, f(x_3^*) = (1.25)^2 = 1.5625, f(x_4^*) = 3.0625$.
   Part (d): $2\pi(1.25)f(1.25)(0.5) \approx 6.136$.
   Part (e): $2\pi x_k^* f(x_k^*)(0.5) = \pi(x_k^*)^3$.
14. $\int_0^2 2\pi y^3\, dy = 2\pi \int_0^2 y(y^2)\, dy$, which represents the volume of the solid of revolution obtained by rotating the region between the graph of $f(x) = \sqrt{x}$, the y-axis, and the line $y = 2$ around the x-axis. (Note that if $x = y^2$ and $y \in [0, 2]$ then $y = \sqrt{x}$.)
16. The region between the graph of $f(x) = \cos x$ and the x-axis on the interval $[0, \frac{\pi}{2}]$, rotated around the y-axis.
24. TYPO: Do with *both* methods (disc/washer method and shell method).
    With shells: $2\pi \int_0^1 (3 - x)(x^2 + 1)\, dx$.
    With discs and washers: $\pi \int_0^1 (3^2 - 1^2)\, dy + \pi \int_1^2 ((3 - \sqrt{y-1})^2 - (1 - \sqrt{y-1})^2)\, dy$.
27. Using the midpoint of each subinterval for y_k^*: $2\pi(.25)(4 - (.25)^2)(.5) + 2\pi(.75)(4 - (.75)^2)(.5) + 2\pi(1.25)(4 - (1.25)^2)(.5) + 2\pi(1.75)(4 - (1.75)^2)(.5) \approx 25.9181$.
31. $2\pi y_k^*(4 - (y_k^*)^2)\Delta y$.
35. Part (a): $\pi \int_0^4 (\sqrt{x})^2\, dx = 8\pi$.
    Part (b): $2\pi \int_0^2 y(4 - y^2)\, dy = 8\pi$.
40. The volume of the k^{th} shell is $2\pi x_k^*(4 - (4 - (x_k^*)^2))\Delta x$, so the volume of the entire solid is $2\pi \int_0^2 x(4 - (4 - x^2))\, dx = 2\pi \int_0^2 x^3\, dx = 8\pi$.
41. $2\pi \int_0^6 x((x - 3)^2 + 2)\, dx = 180\pi$.
42. $2\pi \int_0^1 y(\sqrt{y} - y^2)\, dy = \frac{3\pi}{10}$.
45. TYPO: Should say "rotated around the line $y = 10$."
    $2\pi \int_0^9 (10 - y)(3 - \sqrt{y})\, dy = \frac{657\pi}{5}$.

Section 15.3 (continued)

46. With discs along the x-axis: $\pi \int_0^3 (9-x^2)^2\, dx = \frac{648\pi}{5}$.

47. With shells along the x-axis: $2\pi \int_0^2 x(x^2-4x+4)\, dx = \frac{8\pi}{3}$.

48. With shells along the y-axis: $2\pi \int_0^3 ye^{\frac{y}{2}}\, dy = 2\pi(4+2e^{\frac{3}{2}}) \approx 81.4513$. With discs and washers along the x-axis: $\pi \int_0^1 3^2\, dx + \pi \int_1^{e^{\frac{3}{2}}} (3^2-(2\ln x)^2)\, dx = 9\pi + (-1+4e^{\frac{3}{2}})\pi \approx 81.4513$.

52. The region between the graph of $y = \frac{5}{3}x$ and the lines $y = 5$ and $x = 0$, rotated around the y-axis, is a cone of radius 3 and height 5. The volume of this solid is $2\pi \int_0^3 x(5-\frac{5}{3}x)\, dx = 15\pi$.

53. See the reading.

55. A sphere of radius r can be obtained region between the graph of $y = \sqrt{r^2-x^2}$ and the x-axis on $[-r, r]$ around the x-axis. The volume of this solid is $2\pi \int_0^r x(\sqrt{r^2-x^2} - (-\sqrt{r^2-x^2}))\, dx = 2\pi \int_0^r x(2\sqrt{r^2-x^2}) = \frac{4}{3}\pi r^3$ (using integration by substitution with $u = r^2 - x^2$). Technical note: Usually we can't do the shell method along the x-axis when we've rotated a region around the x-axis; it only works here because the solid is a perfect sphere, and thus is equivalent to rotating the region between the graph of $x = \sqrt{r^2-y^2}$ and the y-axis from $y = -r$ to $y = r$ around the y-axis.

Section 15.4

8. $\Delta y = 1$, $y_0 = 0, y_1 = 1, y_2 = 2, y_3 = 3, y_4 = 4$. The equation of the line defining the cone is $y = \frac{4}{3.5}x$ (so $x = \frac{3.5}{4}y$). With $y_1^* = 0.5, y_2^* = 1.5, y_3^* = 2.5, y_4^* = 3.5$ the work is approximately $(\pi(\frac{3.5}{4}(0.5))^2(1))(62.4)(3.5) + (\pi(\frac{3.5}{4}(1.5))^2(1))(62.4)(2.5) + (\pi(\frac{3.5}{4}(2.5))^2(1))(62.4)(1.5) + (\pi(\frac{3.5}{4}(3.5))^2(1))(62.4)(0.5) \approx 3301.97$ foot-pounds.

9. $\Delta x = 6$, $x_0 = 0, x_1 = 6, x_2 = 12, x_3 = 18, x_4 = 24$. With $x_1^* = 0$, $x_2^* = 6$, $x_3^* = 12$, $x_4^* = 18$ the mass is approximately $(10.5-0.01527(0)^2)(\pi(2)^2(6)) + (10.5-0.01527(6)^2)(\pi(2)^2(6)) + (10.5-0.01527(12)^2)(\pi(2)^2(6)) + (10.5-0.01527(18)^2)(\pi(2)^2(6)) \approx 2586.45$ grams.

12. $\Delta y = 2$, $y_0 = 0, y_1 = 2, y_2 = 4, y_3 = 6, y_4 = 8$. The length of the slices that involve the "ramp" can be determined using similar triangles. With $y_1^* = 1$, $y_2^* = 3$, $y_3^* = 5$, and $y_4^* = 7$ the force is approximately $62.4(10+2.5)(2)(7) + 62.4(10+7.5)(2)(7) + 62.4(30)(2)(7) + 62.4(30)(2)(7) \approx 78624$ pounds.

13. TYPO: No integrals are needed for this problem (or the next two).
$W = (130 \text{ pounds})(6 \text{ feet}) = 780$ foot-pounds.

14. $m = (8.93\frac{\text{grams}}{\text{cm}^3})(20 \cdot 20 \cdot 13 \text{ cm}^3) = 46436$ grams.

15. $F = (62.4\frac{\text{lbs}}{\text{ft}^3})(\pi(\frac{2}{12})^2 \text{ ft}^2)(\frac{7}{12} \text{ ft}) = 3.1765$ pounds.

16. Part (a): $W_{\text{piece}} \approx (62.4)((8)(5)\Delta y)(3-y_k^*)$, so $W = (62.4)(40) \int_0^3 (3-y)\, dy = 11232$ foot-pounds.
Part (b): $m = (62.4)(8 \cdot 5 \cdot 3) = 7488$ pounds.
Part (c): $F_{\text{piece}} \approx (62.4)(8\Delta y)(3-y_k^*)$, so $F = (62.4)(8) \int_0^3 (3-y)\, dy = 2246.4$ pounds.

17. $W_{\text{piece}} \approx (62.4)(\pi(\frac{2}{5}y_k^*)^2\Delta y)(10-y_k^*)$, so $W = (62.4)(\frac{4\pi}{25}) \int_0^{10} y^2(10-y)\, dy = 26138.1$ foot-pounds.

19. Part (a): $W_{\text{piece}} \approx (62.4)(\pi(4)^2\Delta y)(16-y_k^*)$,
so $W = (62.4)(16\pi) \int_0^{13} (16-y)\, dy = 387366$ foot-pounds.
Part (b): $W_{\text{piece}} \approx (71.8)(\pi(4)^2\Delta y)(20-y_k^*)$,
so $W = (71.8)(16\pi) \int_0^{\frac{26}{3}} (20-y)\, dy = 490030$ foot-pounds.

23. TYPO: The density function should be $\rho(x) = 4.2 + 0.4x - 0.03x^2$.
$m_{\text{piece}} \approx (4.2 + 0.4x_k^* - 0.03(x_k^*)^2)((1.5)(1.5)\Delta x$,
so $m = (1.5)^2 \int_0^{12} (4.2 + 0.4x - 0.03x^2)\, dx = 139.32$ grams.

Section 15.4 (continued)

26. Part (a): The parabola described is of the form $y = A(x+10)(x-10)$, and $y(0) = 8$ implies that $A = -\frac{2}{25}$. Therefore $y = -\frac{2}{25}(x+10)(x-10) = 8 - \frac{2}{25}x^2$. Thus the pudding cake is the solid of revolution obtained by revolving the region between the graph of $y = 8 - \frac{2}{25}x^2$ and the x-axis on $[0, 10]$ around the y-axis.
Part (b): $\rho(y)$ is linear, say $\rho(y) = my + b$. Since $\rho(0) = 1.12$ we know that $b = 1.12$; then since $\rho(8) = 0.15$ we know that $m = -\frac{0.97}{8}$. In other words we have $\rho(y) = -\frac{0.97}{8}y + 1.12$.
Part (c): Since $y = 8 - \frac{2}{25}x^2$ we have $x = \sqrt{100 - \frac{25}{2}y}$. Thus
$m_{\text{piece}} \approx (-\frac{0.97}{8}y_k^* + 1.12)(\pi(\sqrt{100 - \frac{25}{2}y_k^*})^2 \Delta y)$, so
$m = \pi \int_0^8 (-\frac{0.97}{8}y + 1.12)(100 - \frac{25}{2}y)\, dy = 1001.12$ ounces.

28. $F_{\text{piece}} \approx (62.4)(2\pi(0.4)\Delta y)(1 - y_k^*)$,
so $F = (62.4)(0.8\pi) \int_0^1 (1-y)\, dy = 78.4142$ pounds.

30. TYPO: I forgot to say that the dam was 200 feet high.
The width that each "slice" extends past the base length of 250 feet on the left and the right can be determined with similar triangles.
$F_{\text{piece}} \approx (62.4)((\frac{75}{200}y_k^* + 250 + \frac{75}{200}y_k^*)\Delta y)(200 - y_k^*)$,
so $F = 62.4 \int_0^{200} (\frac{150}{200}y + 250)(200 - y)\, dy = 3.744 \times 10^8$ pounds.

33. First we must identify the solid as a surface of revolution. One way to do this is to consider the region between the graph of $x = \sqrt{10^2 - y^2}$ and the y-axis from $y = -8$ to $y = 8$ and rotate this region around the y-axis; this is the bottom part of the flask. The radius at the top of the truncated sphere is 6 units (since $\sqrt{10^2 - 8^2} = \sqrt{36} = 6$). Thus the top of the sphere can be obtained by rotating the region between the graph of $x = 6$ and the y-axis from $y = 8$ to $y = 15$ around the y-axis. Now using the disc method along the y-axis, we get a volume of $V = \pi \int_{-8}^{8} (\sqrt{10^2 - y^2})^2\, dy + \pi \int_8^{15} 6^2\, dy = 4745.9$ cubic centimeters.