Homework for Week 2 **Math 232 Spring 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 7.4 1, 3, 5, 9, 10, 12, 18, 19, 20, 23, 24, 25, 28.

Section 8.1 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 17, 20, 22, 23, 25, 26, 29, 30.

Section 8.2 1, 6, 7, 8, 9, 11, 12, 19, 21, 25, 28, 29, 30, 32, 38, 39, 43, 45, 49, 50.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test.

Section 7.4

1. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.
3. $\frac{dV}{dt} = 2r^2 \frac{dE}{dt}$, or equivalently, $\frac{dV}{dt} = \frac{1}{2\pi^2} E^2 \frac{dE}{dt}$.
5. Part (a): $\frac{dV}{dr} = 2\pi rh + \pi r^2 \frac{dh}{dt}$, $\frac{dV}{dh} = 2\pi r \frac{dr}{dt} h + \pi r^2$, $\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$.
   Part (b): $\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$.
   Part (c): $\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h$.
   Part (d): $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$.
   Part (e): $h = 2r$, so $r = \frac{1}{2}h$. Then $\frac{dV}{dt} = \frac{3\pi}{4} h^2 \frac{dh}{dt}$.
9. Part (a): $V = \frac{4}{3}\pi x^3$, $SA = 4\pi x^2$.
   Part (b): $V = \pi y^2 s$, $SA = 2\pi ys + 2\pi y^2$.
   Part (c): $V = \frac{1}{3}\pi v^2 w$, $SA = 4\pi v^2$.
   Part (d): $V = \frac{\pi}{4} h^3$, $SA = \frac{3\pi}{2} h^2$.
   Part (e): $V = \pi r^3$, $SA = \pi(\sqrt{10} + 1)r^2$.
12. $f' = 2uu' + kv'$.
18. $f' = w'(u+t)^2 + 2w(u+t)(u'+1)$.
19. $f' = \frac{1}{k}(u't + u + w')$
20. Part (a): $\frac{dA}{dt}\Big|_{r=24} = 192\pi \frac{\text{in}^2}{\text{sec}}$. Part (b): $\frac{dA}{dt}\Big|_{A=200} = 80\sqrt{2\pi} \frac{\text{in}^2}{\text{sec}}$.
23. Part (a): $\frac{dr}{dt}\Big|_{r=12} = \frac{5}{24\pi} \frac{\text{in}}{\text{sec}}$. Part (b): $\frac{dr}{dt}\Big|_{V=300} = \frac{120}{(4\pi)^{\frac{1}{3}} 900^{\frac{2}{3}}} \frac{\text{in}}{\text{sec}}$. Part (c): $16 \frac{\text{in}^2}{\text{sec}}$.
    Hint on Part (c): You'll need to use volume information to determine $\frac{dr}{dt}\Big|_{r=15}$.
24. Suppose x is Kenneth's distance from the streetlight, l is the length of his shadow, and $y = x + l$ is the distance from the tip of his shadow to the streetlight.
    Part (a): $\frac{dl}{dt}\Big|_{x=10} = -\frac{12}{7} \frac{\text{ft}}{\text{sec}}$. (Note this is actually *decreasing*, not increasing as it says in the problem. Also, you do not need the piece of information that $x = 10$; the rate $\frac{dl}{dt}$ is a constant $-\frac{12}{7} \frac{\text{ft}}{\text{sec}}$ regardless of how far Kenneth is from the streetlight.
    Part (b): $\frac{dy}{dt}\Big|_{x=10} = -\frac{40}{7} \frac{\text{ft}}{\text{sec}}$ (this is in fact also constant, *i.e.* does not depend on x).

Section 7.4 (continued)

25. Part (a): $\frac{dh}{dt}\Big|_{x=4} = -\frac{2}{\sqrt{128}}\frac{ft}{sec}$. Part (b): $\frac{dA}{dt}\Big|_{x=6} = \sqrt{108} - \frac{9}{\sqrt{108}}\frac{\text{ft}^2}{\text{sec}}$.
28. $\frac{dh}{dt}\Big|_{h=3} = -\frac{5}{36\pi}\frac{\text{in}}{\text{sec}}$. (Hint: There are *two* cones in this problem; the ice cream cone (which is a constant size), and the cone of the actual ice cream inside (which changes size). You will need to use similar triangles.)

Section 8.1

2. See the reading.
3. Neither; the base is not a constant, and neither is the exponent.
4. See the reading.
5. $\sqrt{3} \approx 1.73205$. We have $2^{1.7} \approx 3.2490$, $2^{1.73} \approx 3.3173$, $2^{1.732} \approx 3.3219$, $2^{1.7320} \approx 3.3219$, $2^{1.73205} \approx 3.3220$, and so on. Each of these approximations gets closer to the value of $2^{\sqrt{3}}$.
6. $f(3) = 2^9 = 512$ while $g(3) = 8^2 = 64$. The solutions to $2^{(x^2)} = (2^x)^2$ are $x = 0$ and $x = 2$.
7. Since $f(x) = b^x$ is a function, $x = y \;\Rightarrow\; f(x) = f(y)$. Since $f(x) = b^x$ is one-to-one, $f(x) = f(y) \;\Rightarrow\; x = y$.
8. Hint: Use the quadratic formula (where the "x" is 4^x).
9. False.
10. True.
11. False.
13. This is an exponential function: $\frac{3}{2^x} = 3(\frac{1}{2})^x$.
17. This is an exponential function: $5^x 2^{3-x} = 8(\frac{5}{2})^x$.
20. This is not an exponential function; it cannot be written in the form Ab^x.
22. $x = -3$ (where did you use the fact that the function 2^x is one-to-one?).
23. $x = 6$.
25. $x = 0$ and $x = 3$ (hint: cancel a 3^x).
26. $\emptyset$ ($2^x + 1$ is never zero; why?).
29. $x = 0$ and $x = 2$ (put in the form of a quadratic in 5^x).
30. $x = 0$ and $x = -2$ (put in the form of a quadratic in $(\frac{1}{2})^x$).
31. See the reading.
32. See the reading.

Section 8.2

1. $e := \lim_{h\to 0}(1+h)^{\frac{1}{h}}$, so calculate $(1+h)^{\frac{1}{h}}$ for smaller and smaller values of h until the first ten decimal places stabablize. (You can do six decimal places if you calculator doesn't do ten decimal places.)
7. (a) 1; (b) e; (c) DNE; (d) 0; (e) 1.
11. False.
12. False.
19. 6.
21. ex^2.
25. $x = \frac{2}{\ln 3}$.
28. $x = -2$, $x = 3$.
29. $x = \pm\sqrt{\dfrac{e^{\frac{7}{4}} + 3}{2}}$.
30. $x = e^{\frac{\ln 6 - 4}{2}} = \frac{\sqrt{6}}{e^2}$.
32. $f(x) = 2\,e^{(\ln 3)x} \approx 2\,e^{1.0986x}$.

Section 8.2 (continued)

38. $f(x) = \frac{2}{e^4}(e^3)^x \approx 0.03663(20.0855)^x$.

39. $f(x) = 9e^{\frac{\ln(\frac{2}{3})}{2}x} \approx 9e^{-0.2027x}$.

43. $f(x) = 2e^{\frac{\ln(\frac{1}{32})}{5}x} \approx 2e^{-0.6931x}$.

45. See the reading.

49. **Proof:** Given any exponential function $f(x) = Ae^{kx}$ (this assumes that $k \neq 0$), define $b := e^k$ (note that since $k \neq 0$ we have $b \neq 1$; moreover, $b > 0$). Now $f(x) = Ae^{kx} = A(e^k)^x$(by algebra) $= Ab^x$(by our definition of b). ■

50. See the reading.