Homework for Week 3 **Math 232 Spring 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 8.3 1, 2, 7, 8, 16, 17, 21, 22, 26, 32, 33, 39*, 43, 45, 49, 51, 53, 54, 57, 58.

Section 8.4 1, 3, 4, 8, 9, 11, 12, 13, 23, 24, 27, 28, 31, 35, 36, 37, 38, 39, 40, 41.

Section 8.5 2, 3, 4, 6, 7, 9, 11, 14, 15, 16, 19, 20, 23, 31, 35, 37, 39, 40, 41.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 8.3

8. For all $\epsilon > 0$, there exists $\delta > 0$ such that $0 < |x - c| < \delta \Rightarrow |e^x - e^c| < \epsilon$.

26. $\lim\limits_{x\to-\infty} \dfrac{(\frac{1}{2})^x - 1}{(\frac{3}{4})^x} \to \dfrac{\infty - 1}{\infty} = \dfrac{\infty}{\infty}$, which is indeterminate. (See problem 43.)

39. TYPO: The numerator should read $4^x - 6(2^x) + \underline{5}$. (That's why the problem has a "star".)
$\lim\limits_{x\to 0} \dfrac{4^x - 6(2^x) + 5}{1 - 2^x} = \lim\limits_{x\to 0} \dfrac{(2^x - 1)(2^x - 5)}{1 - 2^x} = \lim\limits_{x\to 0}(-2^x + 5) = -2^0 + 5 = -1 + 5 = 4.$

43. $\lim\limits_{x\to-\infty} \dfrac{2^x - 4^x}{3^x} = \lim\limits_{x\to-\infty} ((\frac{2}{3})^x - (\frac{4}{3})^x) \to \infty - 0 \to \infty.$

49. $\lim\limits_{x\to\infty} \dfrac{2e^{1.5x}}{3e^{2x} + e^{1.5x}} = \lim\limits_{x\to\infty} \dfrac{2e^{-0.5x}}{3 + e^{-0.5x}} \to \dfrac{2(0)}{3+0} = 0.$

54. $\lim\limits_{x\to\infty} \dfrac{1}{2 + 3^x} \to \dfrac{1}{2+\infty} = 0$, and $\lim\limits_{x\to-\infty} \dfrac{1}{2+3^x} \to \dfrac{1}{2+0} = \dfrac{1}{2}$;
so $f(x)$ has horizontal asympotes at $y = 0$ on the right and $y = \frac{1}{2}$ on the left.

Section 8.4

8. Without algebra first: $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt{3^x}}\right) = \dfrac{0(\sqrt{3^x}) - 1(\frac{1}{2}(3^x)^{-\frac{1}{2}}(\ln 3)3^x)}{(\sqrt{3^x})^2} = \dfrac{-(\ln 3)3^x}{2\sqrt{3^x}3^x} = -\dfrac{\ln 3}{2\sqrt{3^x}}.$
With algebra first: $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt{3^x}}\right) = \dfrac{d}{dx}\left((3^x)^{-\frac{1}{2}}\right) = \dfrac{d}{dx}\left(3^{-\frac{1}{2}x}\right) = (\ln 3)3^{-\frac{1}{2}x}(-\dfrac{1}{2}) = -\dfrac{\ln 3}{2\sqrt{3^x}}.$

11. We can't do this without using algebra first, since neither the base nor the exponent is constant, so no rules apply. $\dfrac{d}{dx}((2^x)^x) = \dfrac{d}{dx}(2^{x\cdot x}) = \dfrac{d}{dx}\left(2^{(x^2)}\right) = (\ln 2)2^{(x^2)}(2x).$

12. $f(x) = \frac{2}{3}e^{3x} + C$ for any constant C.

13. $f(x) = \frac{1}{\ln 2}\,2^x + C$ for any constant C.

23. $\dfrac{d}{dx}\left(\dfrac{2^{3x}}{x^2 - 1}\right) = \dfrac{(\ln 2)2^{3x}(3)(x^2-1) - 2^{3x}(2x)}{(x^2-1)^2}.$

Section 8.4 (continued)

24. Without algebra first: $\frac{d}{dx}\left(2\sqrt{e^{3x+1}}\right) = 2(\frac{1}{2})(e^{3x+1})^{-\frac{1}{2}}(e^{3x+1})(3)$.
With algebra first: $\frac{d}{dx}\left(2\sqrt{3^{3x+1}}\right) = \frac{d}{dx}\left(2e^{\frac{1}{2}(3x+1)}\right) = \frac{d}{dx}\left(2e^{\frac{3}{2}x+\frac{1}{2}}\right) = 2e^{\frac{3}{2}x+\frac{1}{2}}(\frac{3}{2})$.

31. $f'(x) = (2x)3^x + x^2(\ln 3)3^x = x3^x(2 + (\ln 3)x) = 0$ if $3^x = 0$ (never), or if $x = 0$, or if $2 + (\ln 3)x = 0$ (*i.e.* if $x = \frac{-2}{\ln 3}$). Thus the CPs are $x = 0$ and $x = \frac{-2}{\ln 3}$.

35. $f'(x) = \frac{1}{3}(e^{3x+1} - 2)^{-\frac{2}{3}}(3e^{3x+1}) = \dfrac{e^{3x+1}}{(e^{3x+1} - 2)^{\frac{2}{3}}}$ is zero when $e^{3x+1} = 0$ (never), and DNE when $e^{3x+1} = 2$ (so $\ln 2 = 3x + 1$, and thus $x = \frac{\ln 2 - 1}{3}$). Thus the only CP is $x = \frac{\ln 2-1}{3}$.

36. "...the number of fruit flies in the population."

37. Part (a): If the rate of change is constant, then $r(t)$ is linear. Using $r(0) = 45$ and $r(30) = 53$ to solve for the constants in $r(t) = mt + b$, we get $r(t) \approx 0.2667t + 45$. Technically $t = 99$ is the 100^{th} day of the year, so we find $r(99) \approx 71.4$ rats, or approximately 71 rats.
Part (b): If the rate of change of $r(t)$ is proportional to $r(t)$, then $r(t)$ must be an exponential function. Using $r(0) = 45$ and $r(30) = 53$ to solve for the constants in $r(t) = Ae^{kt}$, we get $r(t) \approx 45e^{0.00545t}$. Then $r(99) \approx 77.2$ rats, or approximately 77 rats.

38. Part (a): Since $A(t)$ increases at a rate proportional to $A(t)$ itself, we know that $A(t)$ must be an exponential function. Using $A(0) = 1000$ and $A(3) = 1260$ to solve for the constants in $A(t) = A_0e^{kt}$, we get $A(t) \approx 1000e^{0.077t}$.
Part (b): $A(30) \approx \$10,074.42$.
Part (c): Solve $A(t) = 4000$ to get $t \approx 18$ years.

Section 8.5

3. If $f(x) = 2(3^x)$ then $f(-x) = 2(\frac{1}{3})^x$ (see Problem 2). Therefore the graph of $y = 2(\frac{1}{3})^x$ can be obtained from the graph of $y = 2(3^x)$ by reflection over the y-axis.

11. $f(x) = -5e^{-x} + 10$ (think about the graphs of $5e^{-x}$ and $-5e^{-x}$ first, then shift up by 10 units). Other functions work too, for example $f(x) = -5(\frac{1}{2})^x + 10$.

14. False.

15. False.

16. True.

19. False.

35. Start with the graph of $y = e^{3x}$, then shift left two units and stretch vertically by a factor of four.

37. Start with the graph of $y = (\frac{1}{2})^x$, then reflect over the x-axis and shift up by 10 units.

40. The domain is $(-\infty, \infty)$, since the denominator is never zero. $f'(x) = \dfrac{60e^{-3x}}{(2e^{-3x} + 5)^2}$ is always positive, so f is always increasing. $f''(x) = \dfrac{-60e^{-3x}(-12e^{-6x} + 75)}{(2e^{-3x} + 5)^4}$ always exists, but is zero at $x = \ln(\frac{75}{2})/(-6) \approx -0.604$. f'' is positive to the left of $x = -0.604$ and negative to the right (so f is concave up and then concave down). Also $f(-0.604) \approx 0.5799$. Finally, $\lim_{x\to\infty} \dfrac{10}{2e^{-3x} + 5} = \dfrac{10}{0 + 5} = 2$, and $\lim_{x\to-\infty} \dfrac{10}{2e^{-3x} + 5} \to \dfrac{10}{\infty + 5} \to 0$, so f has horizontal asymptotes at $y = 2$ on the right and $y = 0$ on the left.