Homework for Week 4 **Math 232 Spring 2001**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 8.6 3, 5, 6, 7, 10, 12, 13, 14, 16, 17, 18, 19.

Section 8.7 2, 4, 6, 8, 9, 11, 12, 18, 24, 26, 30, 34, 36, 38, 39, 40, 45, 48, 51, 52.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 8.6

7. 15% compounded daily is better, since your balance would increase by 16.18% per year (rather than 16% per year).
10. TYPO: The last one should say $Q(34)$.
    $Q(17) = \frac{1}{2}Q_0$, $Q(51) = \frac{1}{8}Q_0$, $Q(-32) = 4Q_0$.
12. Yes, since if $\frac{dW}{dt}$ is proportional to $W(t)$, then $W(t)$ is an exponential function, and all exponential functions have a constant doubling time.
14. (a) 16.98 years; (b) 26.91 years; (c) $\ln(0.96) \approx -0.0408$.
16. Part (a): If yearly, \$98,711.43; if monthly, \$131,571.79; if daily, \$135,428.50; if four times a day, \$135,529.50. (Yes, that's actually true, starting with only \$5000!)
    Part (b): If continuously, \$135,563.19.
    Part (c): If monthly, 1.243597% a year; if daily, 1.245994% a year; if continuously, 1.246077% a year.
    Part (d): About 34.87 years.
17. (a) 12.36 days; (b) 66.14 days; (c) 11.869%; (d) $\ln(\frac{840}{600})/3 \approx 0.11216$.
18. (a) 21.328 hours; (b) 2.5237 hours; (c) 31.607%.
    Part (d): Believe it or not, the answer is 72,800%. There are a number of ways to get this, but in each of the ways the final step involves noticing that $1458 = 2(729) = 2(1 + 728)$ people know the rumor by the end of the first day; remember you have to multiply the "r" in "$1 + r$" by 100 to get the percent increase.
19. (a) 156 years; (b) 53.6 years ago; (c) 0.19233 grams are left, which is 0.0769% of 250 grams; (d) 125.34 years; (e) 2.36%; (f) $\ln(\frac{1}{2})/29 \approx -0.0239$.

Section 8.7

6. $\lim_{x\to\infty} \frac{x}{2^x} \to \frac{\infty}{\infty}$, and $\lim_{x\to\infty} \frac{1/2^x}{1/x} \to \frac{0}{0}$; the first one is easier for L'Hôpital's Rule.
12. Hint: The error is in the second application of L'Hôpital's Rule; why?
18. False.
24. 0 (factor to do without L'H).

Section 8.6 (continued)

26. $-\infty$ (divide numerator and denominator by e^{3x} to do without L'H; you'll get $\frac{1}{0^-}$, since $e^{-3x} < 2e^{-x}$ for large values of x).

30. 1 (use L'H twice).

34. $\lim_{x\to 0} \dfrac{x^{-2}}{x^{-3}+1} = \lim_{x\to 0} \dfrac{\frac{1}{x^2}}{\frac{1}{x^3}+1} = \lim_{x\to 0} \dfrac{x}{1+x^3} = \dfrac{0}{1+0} = 0.$

36. $\lim_{x\to\infty} (e^x - \dfrac{e^x}{x+1})$ (which $\to \infty - \infty$) $= \lim_{x\to\infty} \dfrac{xe^x}{x+1} \overset{\text{L'H}}{=} \lim_{x\to\infty} \dfrac{e^x + xe^x}{1} \to \infty + \infty = \infty.$

38. 0 (rewrite as $\lim_{x\to\infty} \dfrac{x}{2^x}$ and use L'H).

39. $\lim_{x\to\infty} \dfrac{xe^x}{e^{2x}+1} \overset{\text{L'H}}{=} \lim_{x\to\infty} \dfrac{e^x + xe^x}{2e^{2x}} = \lim_{x\to\infty} \dfrac{e^x(x+1)}{2e^{2x}} = \lim_{x\to\infty} \dfrac{x+1}{2e^x} \overset{\text{L'H}}{=} \lim_{x\to\infty} \dfrac{1}{2e^x} \to \dfrac{1}{\infty} \to 0.$

40. $\lim_{x\to\infty} \dfrac{e^{-3x}}{x^2+3x+1}$ (which $\to \frac{0}{\infty}$) $= \lim_{x\to\infty} \dfrac{1}{e^x(x^2+3x+1)} \to \dfrac{1}{\infty\cdot\infty} \to \dfrac{1}{\infty} \to 0.$

45. $\lim_{x\to\infty} \dfrac{(1.1)^x}{50x^3} \overset{\text{L'H}}{=} \lim_{x\to\infty} \dfrac{(\ln 1.1)(1.1)^x}{150x^2} \overset{\text{L'H}}{=} \lim_{x\to\infty} \dfrac{(\ln 1.1)^2(1.1)^x}{300x} \overset{\text{L'H}}{=} \lim_{x\to\infty} \dfrac{(\ln 1.1)^3(1.1)^x}{300} \to \infty,$
so $g(x) = (1.1)^x$ dominates $f(x) = 50x^3$.

48. $\lim_{x\to\infty} \dfrac{1000e^{3x}}{e^{3x}} = \lim_{x\to\infty} 1000 = 1000 \neq \infty$, and $\lim_{x\to\infty} \dfrac{e^{3x}}{1000e^{3x}} = \lim_{x\to\infty} (\frac{1}{1000}) = \frac{1}{1000} \neq \infty,$
so neither function domainates the other.

51. To use domainance we must first write this limit as a limit as $x \to \infty$:
$\lim_{x\to -\infty} 2^x x^{100} = \lim_{x\to\infty} 2^{-x}(-x)^{100} = \lim_{x\to\infty} \dfrac{x^{100}}{2^x} \to 0$, since 2^x domainates x^{100}.

52. See the reading.