Homework for Week 6 **Math 232 Spring 2002**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 10.1 1, 2, 4, 5, 7, 9, 16, 22, 23, 24, 27, 31, 33, 35, 36, 37, 38, 39, 40.

Section 10.2 2, 5, 7, 8, 9, 10, 12, 14, 17, 20, 22, 24, 25, 29, 32, 40, 46, 49, 55, 56, 57, 62, 67, 72, 73, 75.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 10.1

2. They're *all* equal to $\cot\theta$!

7. TYPO: Should say Figure 8.
(a) True; (b) False; (c) True. (All using the fact that opp > adj.)

16. $\csc 44^\circ = \frac{1}{\sin 44^\circ} \approx \frac{1}{0.694658} \approx 1.439557$.

22. $\csc 30^\circ = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$.

23. $\cot 45^\circ = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$. (Note: $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$; use whichever you like best.)

27. TYPO: Should say Figure 12.
If you think of the 30° angle as θ, then 5 is the "adj" side. Now $\tan 30^\circ = \frac{\text{opp}}{5}$, so $\text{opp} = 5\tan 30^\circ = 5(\frac{1}{\sqrt{3}}) = \frac{5}{\sqrt{3}}$. Also, $\frac{5}{\text{hyp}} = \cos 30^\circ$, and thus $\text{hyp} = \frac{5}{\cos 30^\circ} = \frac{5}{\sqrt{3}/2} = \frac{10}{\sqrt{3}}$.

31. $\tan 63^\circ = \frac{x}{10}$, so $x = 10\tan 63^\circ \approx 10(1.96261) = 19.6261$.

33. Isosceles means two sides have the same length (in this case 3). Draw the triangle and split it into two 30–60–90 triangles. Each of these triangles has side lengths 3, $\frac{3}{2}$, and $\frac{3\sqrt{3}}{2}$ (by multiplying the sides of our favorite 30-60-90 triangle by three). Therefore the area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(\frac{3\sqrt{3}}{2} + \frac{3\sqrt{3}}{2})(\frac{3}{2}) = \frac{9\sqrt{3}}{4}$.

35. $\sin 32^\circ = \frac{h}{400}$, so $h \approx 211.97$ feet.

36. Suppose x is *half* of the distance between the stars (so x is the "opposite" side of both triangles). Then $\tan 1^\circ = \frac{x}{60}$, so $x \approx 1.0473$; thus the stars are approximately $2x \approx 2.0946$ light years apart.

37. **Proof (a)**: Suppose θ is an acute angle. Then:
$$\begin{aligned} \csc\theta &= \frac{\text{hyp}}{\text{opp}} && \text{(by definition of } \csc\theta) \\ &= \frac{1}{\text{opp}/\text{adj}} && \text{(algebra)} \\ &= \frac{1}{\sin\theta}. \ \blacksquare && \text{(by definition of } \sin\theta) \end{aligned}$$

40. **Proof:** Suppose a and b are the lengths of the legs of the triangle (with a as the side "opposite" to θ). Then $(\sin\theta)^2 + (\cos\theta)^2 = (\frac{a}{1})^2 + (\frac{b}{1})^2 = a^2 + b^2$, which by the Pythagorean Theorem is equal to $a^2 + b^2 = 1^2 = 1$. $\blacksquare$

Section 10.2

7. If (x, y) is on the unit circle and $y = -\frac{1}{3}$ then by the Pythagorean Theorem we have $x^2 + (-\frac{1}{3})^2 = 1^2$, thus $x = \pm\frac{\sqrt{8}}{3}$. Thus $\cos\theta = x$ is $\frac{\sqrt{8}}{3}$ if θ is in the fourth quadrant, and $\cos\theta = x$ is $-\frac{\sqrt{8}}{3}$ if θ is in the third quadrant. (θ cannot be in the first or second quadrant, since the y-coordinate corresponding to θ is negative.)

10. $\frac{\pi}{6}$ radians (or 30°).

14. True.

15. False.

20. False.

22. True.

24. True.

25. True.

40. Your angle should terminate in the fourth quadrant, a little bit after $\frac{3\pi}{2}$ radians (since 5 radians is just a bit more than $\frac{3\pi}{2} \approx 4.71239$ radians).

46. The reference triangle is a 30–60–90 triangle in the third quadrant, with a reference angle of 60°.

49. $\frac{201\pi}{2} = 100\pi + \frac{\pi}{2} = 50(2\pi) + \frac{\pi}{2}$ terminates at the same place as $\frac{\pi}{2}$ radians (so no reference triangle exists).

55. The reference triangle for $-\frac{\pi}{4}$ is a 45–45–90 triangle in the fourth quadrant, with a reference angle of 45°. The signed side lengths of the reference triangle are 1 (hyp), $\frac{\sqrt{2}}{2}$ (horizontal), and $\frac{\sqrt{2}}{2}$ (vertical); in other words, $-\frac{\pi}{4}$ corresponds to the point $(x, y) = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ on the unit circle. Therefore $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$.

67. $\csc\frac{11\pi}{7} = \frac{1}{\sin\frac{11\pi}{7}} \approx -1.0257$. This makes sense because $\frac{11\pi}{7}$ is in the fourth quadrant, just a tiny further than $\frac{3\pi}{2}$. Thus the y-coordinate corresponding to $\frac{11\pi}{7}$ should be just a little bit more than -1 (closer to zero is "more", since the y-coordinate is negative); therefore $\csc\frac{11\pi}{7} = \frac{1}{y}$ should be a negative number that is just a little bit *less* than -1 (like -1.0257, for example).

72. $\sin\theta = \frac{1}{5}$ means that θ corresponds to a point $(x, \frac{1}{5})$ on the unit circle, for some x. (Visually, the point on the unit circle corresponding to θ is one of the two points on the unit circle whose height is $\frac{1}{5}$; draw a picture!) Since $\frac{\pi}{2} < \theta < \pi$, we know that θ terminates in the second quadrant, and thus that the x-coordinate is negative. By the Pythagorean Theorem (or, if you like, by the fact that (x, y) is on the unit circle), we have $x^2 + (\frac{1}{5})^2 = 1^2$; thus $x = \pm\frac{\sqrt{24}}{5}$. Since we know x is negative, we must have $x = -\frac{\sqrt{24}}{5}$. Therefore $\cos\theta = x = -\frac{\sqrt{24}}{5}$.