Homework for Week 7 **Math 232 Spring 2002**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 10.3 3, 6, 9, 16, 19, 27, 29, 30, 33, 37, 39, 42, 45, 46, 50, 56, 59, 61, 64, 67.

Section 10.4 1, 2, 7, 10, 14, 15, 16, 29, 31, 35, 37, 42, 48, 49, 50, 52, 53*, 54, 59.

Section 10.5 3, 4, 5, 6, 11, 15, 21, 22, 23, 26, 30, 34*, 37, 43, 44, 45, 49, 52, 54, 57.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 10.3

3. Hint: $\sqrt{3} = \frac{\sqrt{3}/2}{1/2}$.
30. Hint: The algebra in the proof is right. Explain why the logic is wrong.
33. $\frac{\pi}{3} + 2\pi k$ and $\frac{5\pi}{3} + 2\pi k$, for any positive or negative integer k.
39. $\sec\theta \geq 0$ implies that θ is in the first or fourth quadrant; therefore the domain is the set $\ldots \cup [-\frac{5\pi}{2}, -\frac{3\pi}{2}] \cup [-\frac{\pi}{2}, \frac{\pi}{2}] \cup [\frac{5\pi}{2}, \frac{7\pi}{2}] \cup \ldots$; in other words, the domain is every interval of the form $[-\frac{\pi}{2} + 2\pi k, \frac{\pi}{2} + 2\pi k]$ where k is a positive or negative integer.
45. $f(-\theta) = \frac{\tan(-\theta)}{\csc(-\theta)+\cot(-\theta)} = \frac{-\tan\theta}{-\csc\theta-\cot\theta} = -f(\theta)$ (since tangent, cosecant, and cotangent are each odd functions). Thuse $f(\theta)$ is an odd function.
50. Hint: $\frac{7\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$; use the sum identity for cosine, then use the unit circle to determine the values of the trigonometric functions.
56. Mimic the proof of Theorem 3a.
59. Mimic the proof of Theorem 5a.
61. **Proof:** (part (a)) By Theorem 5c, $\frac{1-\cos 2\theta}{2} = \frac{1-(1-2\sin^2\theta)}{2} = \frac{2\sin^2\theta}{2} = \sin^2\theta$. ■
64. **Proof:** First of all, $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}} = \frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$.

    On the other hand, by Theorem 5a and 5b, $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$.

    Thus $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ (since these two expressions are equal to the same thing). ■

Section 10.4

7. Hint: (a) shift up 2 units; (b) shift left 2 units; (c) vertical stretch by a factor of 2; (d) horizontal squish by a factor of 2.
10. $\lim_{x\to 0}(\cos x - 1) = 0$ implies that $\lim_{x\to 0}\cos x = 1$ (why?); therefore $\lim_{x\to 0}\cos x = \cos 0$, so $\cos x$ is continuous at $x = 0$.
14. Hint: Think about $\frac{\text{bounded}}{\infty}$.
15. Hint: Think about $\frac{\infty}{\text{bounded}}$; but watch out! The denominator is sometimes positive and sometimes negative!
16. False.
29. $\lim_{x\to 0} \frac{1-\cos 2x}{7x} = \lim_{x\to 0}(\frac{2}{7})\frac{1-\cos 2x}{2x} = (\frac{2}{7})(0) = 0.$

Section 10.4 (continued)

31. $\lim_{x\to 0} \frac{\sin^2 3x}{x^3-x} = \lim_{x\to 0} \left(\frac{\sin 3x}{3x} \cdot \frac{3\sin 3x}{x^2-1}\right) = 1(\frac{0}{1}) = 0.$

35. $\lim_{x\to 0} \frac{x^2 \csc 3x}{1-\cos 2x} = \lim_{x\to 0} \left(\frac{2x}{1-cos2x} \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{6}\right) = 0(1)(\frac{1}{6}) = 0.$

49. $\lim_{x\to 0} \sin(\frac{1}{x}) \to \sin(\infty)$, which does not exist.

50. $\lim_{x\to 0} x\sin(\frac{1}{x}) \to 0(\text{bounded}) = 0.$

52. Part (a): The limit DNE, since both sine and cosine oscillate between -1 and 1 as $x \to \infty$.
Part (b): There is no friction to damp the oscillations of the spring.

53. Part (a): $\lim_{t\to\infty} s(t) = \lim_{t\to\infty} \dfrac{A\sin(\frac{\sqrt{4km-f^2}}{2m}\,t) + B\cos(\frac{\sqrt{4km-f^2}}{2m}\,t)}{e^{\frac{f}{2m}\,t}} \to \dfrac{\text{bounded}}{\infty} \to 0.$
Part (b): TYPO: the last part of the equation should be $B\cos\left(\frac{\sqrt{4km-f^2}}{2m}\,t\right)$.
Friction causes each oscillation to be smaller than the last.

54. See the reading.

59. See the reading.

Section 10.5

4. If x is in degrees, then the slope of the graph of $\sin x$ at $x = 0$ is very small, and in particular not equal to $\cos 0 = 1$. To convince yourself of this, sketch $\sin x$ (in degrees) together with the line $y = x$ (which has slope 1 at $x = 0$).

5. Bad differentiation in the denominator when applying L'H; also $\frac{-\sin\frac{\pi}{2}}{-\cos\pi} = \frac{-(1)}{-(-1)} = -1.$

11. $f'(x) = \sec(x^3) + x\sec(x^3)\tan(x^3)(3x^2).$

15. $f'(x) = \frac{\cos x}{\sin x}.$

21. $f'(x) = \cos(\cos(\sec x))(-\sin(\sec x))(\sec x\tan x).$

22. $f'(x) = 2\csc(e^x)(-\csc(e^x)\cot(e^x))(e^x).$

23. $f'(x) = e^{\csc^2 x}(2\csc x)(-\csc x\cot x).$

26. Hint: Rewrite as $f(x) = \frac{\sin x}{\cos^2 x}$.

30. Part (a): $\lim_{x\to 0} \frac{\sin^2 3x}{2x^2} = \lim_{x\to 0} \left(\frac{\sin 3x}{3x} \cdot \frac{\sin 3x}{3x} \cdot \frac{9}{2}\right) = (1)(1)(\frac{9}{2}) = \frac{9}{2}.$
Part (b): $\lim_{x\to 0} \frac{\sin^2 3x}{2x^2} \overset{\text{L'H}}{=} \lim_{x\to 0} \frac{2\sin 3x(\cos 3x)(3)}{4x} \overset{\text{L'H}}{=} \lim_{x\to 0} \frac{6(\cos 3x)(\cos 3x)(3)-6(\sin 3x)(-\sin 3x)(3)}{4} =$
$\lim_{x\to 0} \frac{18\cos^2 3x+18\sin^2 3x}{4} = \frac{18(1)+18(0)}{4} = \frac{9}{2}.$

34. TYPO: Just do part (b).
Hint: This limit $\to \infty - \infty$, which is indeterminate; rewrite $\cot x - \csc x$ as $\frac{\cos x - 1}{\sin x}$.

37. 1 (use L'H twice).

43. $\lim_{x\to 0^+} x\ln(\sin x)\ [\to 0(-\infty)] = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}}\ [\to \frac{-\infty}{\infty}] \overset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(-\frac{1}{x^2}\right)}$
$= \lim_{x\to 0^+} \frac{-x^2\cos x}{\sin x}\ [\to \frac{0}{0}] \overset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{-2x\cos x + x^2\sin x}{\cos x} = \frac{0+0}{1} = 0.$

44. 1.

45. $\lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)}{\cos x}\ [\to \frac{0}{0}] \overset{\text{L'H}}{=} \lim_{x\to\frac{\pi}{2}} \frac{\cos(\cos x)(-\sin x)}{-\sin x} = \lim_{x\to\frac{\pi}{2}} \cos(\cos x) = \cos(0) = 1.$

52. Mimic the proof of Theorem 1a.

54. Mimic the proof of Theorem 2b.

57. **Proof:** $\frac{d}{dx}(\cos x) = \frac{d}{dx}(\sin(x + \frac{\pi}{2})) = \cos(x + \frac{\pi}{2})(1) = \cos x\cos\frac{\pi}{2} - \sin x\sin\frac{\pi}{2}$
$= (\cos x)(0) - (\sin x)(1) = -\sin x$. The first equality follows from the identity $\cos x = \sin(x + \frac{\pi}{2})$; the second equality uses the chain rule and the derivative of sine; the third equality uses the sum identity for cosine. ■