Homework for Week 8 **Math 232 Spring 2002**

This homework will not be collected. It is your responsibility to do as many problems as necessary to understand the material (this includes doing extra problems if you need more practice). We recommend that you read each section before attempting any exercises. Next week's quiz will be a subset of the problems below.

Section 10.6 11, 12, 13, 14, 15, 17, 18, 22, 27, 34, 39, 40, 49, 54, 55, 58, 61, 74, 75, 76, 77, 78.

Section 11.1 3, 4, 6, 10, 13, 14, 15, 16, 17, 20, 23, 25, 26, 30, 35, 38, 44, 50, 55, 59, 62, 65.

Section 11.2 1, 3, 7, 8, 9, 12, 15, 16, 20, 21, 24, 26, 27, 28, 35, 36, 38, 39, 41, 43.

Selected Hints and Answers

Caution: The hints and answers below are not necessarily full solutions. Many of them would not be considered complete on a quiz or test. **Answers are not provided for problems whose answers can be found in the reading or problems whose answers are easy to check using a calculator.**

Section 10.6

14. $(2,8)$, $(10,8)$, and $(-6,8)$.

15. $(\frac{\pi}{2},-2)$, $(\frac{3\pi}{2},-2)$, and $(-\frac{\pi}{2},-2)$.

17. The IPs occur at $x=\ldots,-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{3}{2},\ldots$, *i.e.* at $x=\frac{1}{2}+k$ for k any integer. (Graph the function by hand; you know the IPs occur halfway between the maximums and minimums.)

22. One way is $\frac{\pi}{4}+\frac{\pi}{2}k$, where k is any integer.

27. One way is $\frac{\pi}{2}k$, where k is any integer.

34. False.

39. $\frac{\pi}{2}$.

40. 1.

58. $f(x)=3\sin(\frac{1}{2}x)+2$ and $f(x)=3\cos(\frac{1}{2}(x-\pi))+2$ are possible answers.

74.
- The domain of $f(x)$ is $(-\infty,\infty)$.
- $f(x)=e^x\sin x$ is zero if $x=\pi k$ for any integer k.
- $f'(x)=e^x(\sin x+\cos x)$ always exists, and is zero only if x is an angle that terminates at $\frac{3\pi}{4}$ or $-\frac{\pi}{4}$, *i.e.* if $x=\frac{3\pi}{4}+\pi k$ for any integer k.
- $f''(x)=2e^x\cos x$, which always exists, and is zero at $x=\frac{\pi}{2}+\pi k$ for any integer k.
- From the information above, make number lines for f, f', and f''.
- $\lim_{x\to\infty} e^x\sin x \to (\infty)(\text{bounded}) = \text{DNE}$.
- $\lim_{x\to-\infty} e^x\sin x \to (0)(\text{bounded}) = 0$.
- Now calculate some of the values of $f(x)$ at the "important" points, and sketch a graph.

76. Part (a): $P(t)=150\sin(\frac{2\pi}{365}(x-\frac{365}{4})+350$.
Part (b): $P(t)=150\cos(\frac{2\pi}{365}(x-\frac{365}{2})+350$.

77. Part (a): Average high tide is 7.175 feet; average low tide is 0.4 feet; average time between high tides is 1459 minutes; average time between low tides is 1458 minutes.
Part (b): Amplitude is 3.3875, period is approximately 1458.5 minutes. One possible "center" point is $(364, 3.7875)$, corresponding to a negative value of $A=-3.3875$. Then $H(t)=-3.3875\sin(\frac{2\pi}{1458.5}(x-364))+3.7875$ should be a good model (test it!).

Section 11.1

10. $y = \cos^{-1} x$ is an angle. $x = \cos y$ is the horizontal coordinate of the point on the unit circle corresponding to the angle y.
15. Only (a) and (c) are defined.
16. Hint: Use the unit circle and the ranges of the inverse trigonometric functions.
17. $[0, \pi]$, cosine.
23. fourth.
25. $\sec^{-1} x = \theta$ means that $x = \sec\theta = \frac{1}{\cos\theta}$, so $\cos\theta = \frac{1}{x}$.
26. $\frac{1}{3x}$.
35. False.
38. False.
50. The angle $\theta = \arcsin(-\frac{\sqrt{3}}{2})$ must be in the fourth quadrant with a vertical coordinate on the unit circle of $-\frac{\sqrt{3}}{2}$; this corresponds to a 30–60–90 reference triangle and $\theta = -\frac{\pi}{3}$.
55. If $\theta = \text{arcsec}(-\frac{2}{\sqrt{2}})$ then $\cos\theta = -\frac{\sqrt{2}}{2}$ and θ is in the second quadrant; this corresponds to a 45–45–90 reference triangle and an angle of $\theta = \frac{3\pi}{4}$.
65. $\theta = \sec^{-1}(-5)$ means that $-5 = \sec\theta = \frac{1}{\cos\theta}$, so $\cos\theta = -\frac{1}{5}$. Therefore $\theta = \cos^{-1}(-\frac{1}{5}) \approx 1.77215$.

Section 11.2

7. The range of $\sin^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
9. Hint: Think about similar triangles.
12. Let $\theta = \cos^{-1} x$, so that $\cos\theta = x$. Then make a triangle with angle θ that has an adacent side of length x and a hypotenuse of length 1 (why?). Then using the Pythagorean theorem, $\sin(\cos^{-1} x) = \sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$.
15. If $\theta = \sec^{-1} x$ then $\cos\theta = \frac{1}{x}$; make a triangle with angle θ that has adj $= 1$ and hyp $= x$. Then $\tan\sec^{-1} x = \tan\theta = \sqrt{x^2-1}$.
16. $\sec(\cos^{-1} x) = \frac{1}{\cos(\cos^{-1} x)} = \frac{1}{x}$ (restricted to $x \in [-1, 1]$; why?).
20. $f'(x) = \frac{-2}{\sqrt{1-4x^2}}$.
21. $f'(x) = \frac{2\sin 2x}{\cos^2 2x}$.
24. $f'(x) = \frac{6x}{\sqrt{1-9x^4}}$.
26. $f'(x) = \frac{e^x}{|e^x|\sqrt{(e^x)^2-1}} = \frac{1}{\sqrt{e^{2x}-1}}$.
27. $f'(x) = 2x\arctan x^2 + x^2\left(\frac{2x}{1+x^4}\right)$.
35. $f(x) = \frac{2}{9}\tan^{-1}(3x)$.
36. $f(x) = \frac{1}{\sqrt{3}}\sin^{-1}(\sqrt{3}x)$.
38. Tricky: First rewrite $f'(x) = \frac{1}{4+x^2} = \frac{1}{4(1+\frac{1}{4}x^2)} = (\frac{1}{4})\frac{1}{1+(\frac{x}{2})^2}$. Then you can use guess-and-check to find $f(x) = \frac{1}{4}(2)\tan^{-1}\frac{x}{2} = \frac{1}{2}\tan^{-1}\frac{x}{2}$.
39. $f(x) = \ln(1+x^2)$.
41. See the reading.
43. See the reading.