TEST I

CIRCLE ONE: 01 / 02

Math 232
February 12, 2002

Name: _____

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).

- When you do use your calculator, sketch all relevant graphs and explain how you use them.

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements, and add two points for drawing a fish on the scrap page.

- This test has 5 problems and is worth 100 points. Make sure that you have all of the pages!

- Good luck!

1. (*28 points*) Fill in the blanks. No work need be shown. Two points each.

(a) $b^x = b^y \implies x = y$ because the function $f(x) = b^x$ is _____ .

(b)   If $e^{kx} = b^x$ and $k \in (0, \infty)$, then b is in the **interval**: _____

(c)   The natural exponential function is continuous at $x = 0$. Write this fact as a limit statement:

_____

(d)   If a credit card balance $Q(t)$ is charged a 22.99% APR, compounded monthly, write a formula for $Q(t)$:

_____

(e)   By definition, f "dominates" g if: _____

(f)   Which is bigger for $x > 1$: $y = \log_2 x$, $y = \log_4 x$, or neither? _____

(g)   List a condition that would guarantee that a function $Q(t)$ is exponential.

_____

(h) $f(x) = \dfrac{2}{3 + 5^x}$ has a horizontal asymptote on the left at $y = $ _____ .

(i)   If you reflect the function $f(x) = 3^x$ over the y-axis, what function do you get? (Write your answer in the form Ab^x.)

_____

(j)   If you reflect the function $f(x) = 3^x$ over the line $y = x$, what function do you get?

_____

(k) $\lim\limits_{x \to \boxed{}} \log_{\frac{1}{2}} x = \infty$. What goes in the box? _____

(l)   If $f(2) = 4$ and $\dfrac{d}{dx}\Big|_{x=2} (\ln |f(x)|) = -3$, then $f'(2) = $ _____ .

(m)   Circle yes or no and fill in the blanks if "no":

    • The form 0^∞ is indeterminate. (YES) (NO, it approaches _____)

    • The form 1^∞ is indeterminate. (YES) (NO, it approaches _____)

2. (*24 points*) The following questions all have to do with derivatives. Show all work and put boxes around your final answers.

a. (*6 pts*)   Find the derivative of the function $f(x) = \dfrac{2^{3x}}{x^2 - 1}$.

b. (*6 pts*)   Find a function whose **derivative** is $f'(x) = \dfrac{3}{2x + 1}$.

c. (*6 pts*)   Find $\frac{dy}{dx}$ given that $\sqrt{3y - 1} = 5xy$.

d. (*6 pts*)   Find $\frac{d}{dx}\left(x^{\ln x}\right)$.

3. (*6 points*) Dr. Drosophila is breeding fruit flies in his laboratry. He started with 600 fruit flies, and after three days there were already 840 fruit flies in Dr. Drosophila's lab. Assuming that the fruit fly population grows at fixed percentage each day, what is that fixed percentage?

4. (*18 points*) Yes! Algebra problems! Show all work and put your final answers in the boxes. **Do _not_ use a calculator for anything except to find the approximations in part (c)!**

   a. (*6 pts*)   Find the *exact* values of the following quantites.

      • $\log_{1.729} 1 + \ln(\frac{1}{e^4}) = $ ☐

      • $4 \log_2 6 - 2 \log_2 9 = $ ☐

      • $\dfrac{\log_7 9}{\log_7 \frac{1}{3}} = $ ☐

   b. (*6 pts*)   Find the domain of the function $f(x) = \dfrac{\ln(x+1)}{\ln(x-2)}$.

Domain, in interval notation, is: ☐

   c. (*6 pts*)   If $f(x) = 2e^{3x-4} = Ab^x$, find exact *and* approximate values for A and b.

      • $A = $ ☐ $\approx$ ☐

      • $b = $ ☐ $\approx$ ☐

5. (*24 points*) Find each of the following limits. Show all work, and put boxes around your final answers.

a. (*6 pts*) $\displaystyle\lim_{x\to\infty} \frac{1}{2 + (0.7)^x}$

b. (*6 pts*) $\displaystyle\lim_{x\to 0^+} \frac{x}{\log_2 x}$

c. (*6 pts*) $\displaystyle\lim_{x\to\infty} \frac{xe^x}{e^{2x} + 1}$

d. (*6 pts*) $\displaystyle\lim_{x\to 1^+} x^{\frac{1}{x-1}}$

Survey Questions: (worth 2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SCRAP WORK