TEST II

CIRCLE ONE: 01 / 02

Math 232
March 19, 2002

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Circle or otherwise indicate your final answers.
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements, and add two points for drawing a house on the scrap page.
- This test has 8 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

Various Formulas and Identities You May or May Not Need

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}.$$

$$\sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}.$$

$$\sum_{k=1}^{N} k^3 = \frac{N^2(N+1)^2}{4}.$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$E = mc^2$$

1. (*25 points*) Fill in the blanks below.

a. (*2 pts*) $f(x) = \csc x$ has domain _____ and range _____ .

b. (*2 pts*) $f(x) = \arccos x$ has domain _____ and range _____ .

c. (*2 pts*) Suppose a right triangle has angles of 30°, 60°, and 90°, and that the shortest leg of the triangle is five units long. Then the length of the hypotenuse of this triangle is _____, and the length of its longest side is _____.

d. (*2 pts*) The period of $f(x) = 3\cot(\pi - 2x)$ is _____ .

e. (*2 pts*) Which is the largest: **(A)** an angle of $\frac{\pi}{4}$ radians, **(B)** an angle of one radian, or **(C)** the reference angle for $\frac{5\pi}{6}$? _____.

f. (*2 pts*) $\lim_{x\to\infty} \dfrac{e^x}{\sin x} =$ _____ .

g. (*3 pts*) Write the sum $2 + \dfrac{2}{4} + \dfrac{2}{9} + \dfrac{2}{16} + \dfrac{2}{25}$ in sigma notation, starting with $k = 3$:

_____

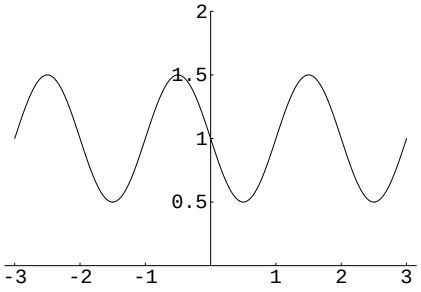
h. (*3 pts*) If $\sum_{k=1}^{4} a_k = 6$, $\sum_{k=0}^{4} b_k = 10$, and $a_0 = 2$, then $\sum_{k=0}^{4}(2a_k + 3b_k) =$ _____ .

i. (*3 pts*) If $\int_3^0 f(x)\,dx = -2$, $\int_3^4 f(x)\,dx = 1$, and $\int_0^4 g(x)\,dx = 5$, then $\int_0^4 (f(x) + 2g(x))\,dx =$ _____ .

j. (*4 pts*) If $\sum_{k=0}^{2} \ln(5 + \frac{k}{3})(\frac{1}{3})$ is a Left Hand Sum, then we must have:
$f(x) =$ _____ , $N =$ _____ , $\Delta x =$ _____ , $a =$ _____ , and $b =$ _____ .

2. (*8 points*) Use the sum formula for cosine and a Pythagorean identity to prove the trigonometric identity $\cos(2\theta) = 1 - 2\sin^2\theta$.

3. (*10 points*) Consider the graph of $f(x)$ shown below.



a. (*5 pts*) Find a function of the form $f(x) = A\sin(B(x+C)) + D$ that has this graph.

b. (*5 pts*) Find a function of the form $f(x) = A\cos(B(x+C)) + D$ that has this graph.

4. (*10 points*) Find $\lim_{x \to 0} \dfrac{\sin^2 3x}{2x^2}$ two ways:

a. (*5 pts*) Using L'Hôpital's Rule.

b. (*5 pts*) Without using L'Hôpital's Rule.

5. (*10 points*) Find the derivatives below. Do ***not*** simplify your answers.

a. (*5 pts*) $\frac{d}{dx}(\sec(\tan x))$

b. (*5 pts*) $\frac{d}{dx}(\sin^{-1}(1 + x - x^2))$

6. (*7 points*) Find the domain of the function $f(x) = \dfrac{x}{1+\cos x}$.

7. (*15 points*) Calculate each of the following values exactly. Do not use a calculator. Your work in each problem should be supported by an angle shown in the unit circle, and a triangle with side lengths written on it.

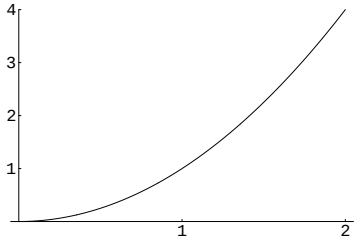
a. (*5 pts*) Find $\sin(-\frac{7\pi}{6})$.

b. (*5 pts*) Find $\sec^{-1}(-2)$.

c. (*5 pts*) Given that $\sin\theta = \frac{1}{5}$ and $\tan\theta < 0$, find $\cos\theta$.

8. (*15 points*) Consider the definite integral $\int_0^2 x^2\,dx$.

a. (*3 pts*) Approximate this definite integral using a Right Hand Sum with $N = 4$ rectangles. Draw the rectangles on the graph below.



b. (*3 pts*) Use sigma notation to write down the general Right Hand Sum with N rectangles for the definite integral above. Write the sum so that the only letters that appear in the sum are k and N. (Note: N is ***not*** equal to 4 here; your answer should include the letter N.)

c. (*3 pts*) Write the sum above as an expression in terms of N (*i.e.* as a simple formula without a "sigma"; you may need a formula from the front page).

d. (*3 pts*) Use your answer above to approximate the definite integral above with $N = 1000$ rectangles.

e. (*3 pts*) Find the *exact* value of the definite integral by taking the limit as $N \to \infty$ of the expression you found above.

Survey Questions: (worth 2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SCRAP WORK