TEST III

CIRCLE ONE: 01/02

Math 232 April 23, 2002

Name:

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

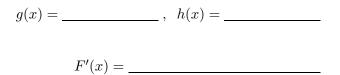
- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- If you run out of room for your work you may continue on the scrap page; however, make sure that you direct me to your work.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements, and add two points for drawing a tree on the scrap page.
- Don't spend too much time on any one problem! Do the ones you that you find easiest first, then move on to the harder problems.
- This test has 5 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (26 points) Circle true or false, fill in the blanks, or answer each question, as appropriate. You do not need to show any work for this problem.

a. (2 pts) T F If
$$u = x^2 + 1$$
 then $du = 2x$.
b. (2 pts) T F $\int_0^3 x \sin x \, dx = -x \cos x + \int_0^3 \cos x \, dx$.
c. (2 pts) T F The average value of a function $f(x)$ on an interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.
d. (2 pts) T F If $f(x) - g(x) = 2$ then $f(x)$ and $g(x)$ have the same derivative.
e. (2 pts) T F If $f(x) - g(x) = 2$ then $f(x)$ and $g(x)$ have the same derivative.
e. (2 pts) T F The Fundamental Theorem of Calculus implies that $\int_a^b f''(x) \, dx = [f'(x)]_a^b$.
f. (2 pts) T F $\int \frac{1}{1 + x + x^2} \, dx = \ln|1 + x + x^2| + C$.
g. (2 pts) T F The integral $\int \frac{x}{\sqrt{9 - x^2}} \, dx$ is best done with a trigonometric substitution.

h. (4 *pts*) State the Fundamental Theorem of Calculus. (Don't forget that a theorem has a hypothesis and a conclusion; you should write something in the form "If ..., then ...".)

i. (4 pts) The function $F(x) = \int_3^{x^2} \sin t \, dt$ is a composition of functions g(h(x)). What are the functions f(x) and g(x)? Then find the derivative F'(x).



j. (4 pts) Consider the function $A(x) = \int_0^x (2-t) dt$. List the following quantities in order from smallest to largest: A(2), A(3), A(4), A'(4), and A''(4). (Hint: A graph of f(t) = 2-t may help, and you may have to calculate A'(x) and A''(x).)

2. (18 points) Solve each of the following integrals. Show your work clearly and put your final answers in the boxes provided.

a. (6 pts) $\int \ln x \, dx$ (Show your work; don't just use the formula.)

b. (6 pts)
$$\int 2\pi x (8 - x^{\frac{3}{2}}) dx$$

c. (6 pts)
$$\int \sec^4 x \tan^4 x \, dx$$

3. (18 points) Solve each of the following integrals. Show your work clearly and put your final answers in the boxes provided.

a. (6 pts)
$$\int_{2}^{e} (x \ln x)^{-1} dx$$

b. (6 pts)
$$\int \left(1 + \frac{1}{e^x}\right)^2 dx$$

c. (6 pts)
$$\int \frac{1}{(x^2+4)^{\frac{3}{2}}} dx$$



4. (20 points) For each problem below, express the quantity described in terms of definite integrals. Use the boxes provided, and show work and/or graphs to earn partial credit. Your answers should not involve the letters f and/or g. DO NOT SOLVE THE INTEGRALS!

a. (4 *pts*) The signed area between the graph of $f(x) = 9 - x^2$ and the x-axis on [0, 4].

b. (4 pts) The "true" area between the graph of $f(x) = 9 - x^2$ and the x-axis on [0, 4].

c. (4 pts) The area between the graphs of $f(x) = 9 - x^2$ and g(x) = 7 - x on [1,5].

d. (4 *pts*) The average value of the function $f(x) = 9 - x^2$ on [-3, 3].

e. (4 pts) The arc length of the graph of $f(x) = 9 - x^2$ on [-2, 1].

5. (18 points) For each problem below, express the quantity described in terms of definite integrals. Use the boxes provided, and show work and/or graphs to earn partial credit. You answers should not involve the letter f. DO NOT SOLVE THE INTEGRALS!

a. (6 pts) The volume of the solid of revolution obtained by rotating the region between the graph of $f(x) = x^2$ and the x-axis from x = 0 to x = 2 around the y-axis, using discs and/or washers.

b. (6 pts) The volume of the solid of revolution obtained by rotating the region between the graph of $f(x) = x^2$ and the x-axis from x = 0 to x = 2 around the line x = 3, using shells.

c. (6 pts) The work required to pump all of the water out of the top of a cylindrical tank that has a radius of 2 feet and a height of 5 feet. (Hint: The work W involved in lifting an object with weight F through a distance d is W = Fd, and the weight of water is 62.4 pounds per cubic foot.)

Survey Questions: (worth 2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SCRAP WORK