TEST III

CIRCLE ONE: 01 / 02

Math 232
April 23, 2002

Name: _____
By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. I will take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- When you do use your calculator, sketch all relevant graphs and explain how you use them.
- If you run out of room for your work you may continue on the scrap page; however, make sure that you direct me to your work.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements, and add two points for drawing a tree on the scrap page.
- Don't spend too much time on any one problem! Do the ones you that you find easiest first, then move on to the harder problems.
- This test has 5 problems and is worth 100 points. Make sure that you have all of the pages!
- Good luck!

1. (*26 points*) Circle true or false, fill in the blanks, or answer each question, as appropriate. You do not need to show any work for this problem.

a. (*2 pts*) **T** **F** If $u = x^2 + 1$ then $du = 2x$.

b. (*2 pts*) **T** **F** $\displaystyle\int_0^3 x \sin x \, dx = -x \cos x + \int_0^3 \cos x \, dx.$

c. (*2 pts*) **T** **F** The average value of a function $f(x)$ on an interval $[a, b]$ is $\dfrac{f(b) - f(a)}{b - a}$.

d. (*2 pts*) **T** **F** If $f(x) - g(x) = 2$ then $f(x)$ and $g(x)$ have the same derivative.

e. (*2 pts*) **T** **F** The Fundamental Theorem of Calculus implies that $\displaystyle\int_a^b f''(x) \, dx = [\, f'(x) \,]_a^b$.

f. (*2 pts*) **T** **F** $\displaystyle\int \frac{1}{1 + x + x^2} \, dx = \ln|1 + x + x^2| + C.$

g. (*2 pts*) **T** **F** The integral $\displaystyle\int \frac{x}{\sqrt{9 - x^2}} \, dx$ is best done with a trigonometric substitution.

h. (*4 pts*) State the Fundamental Theorem of Calculus. (Don't forget that a theorem has a hypothesis and a conclusion; you should write something in the form "If ..., then ...".)

i. (*4 pts*) The function $F(x) = \int_3^{x^2} \sin t \, dt$ is a composition of functions $g(h(x))$. What are the functions $f(x)$ and $g(x)$? Then find the derivative $F'(x)$.

$g(x) =$ _____ , $h(x) =$ _____

$F'(x) =$ _____

j. (*4 pts*) Consider the function $A(x) = \int_0^x (2 - t) \, dt$. List the following quantities in order from smallest to largest: $A(2)$, $A(3)$, $A(4)$, $A'(4)$, and $A''(4)$. (Hint: A graph of $f(t) = 2 - t$ may help, and you may have to calculate $A'(x)$ and $A''(x)$.)

_____ < _____ < _____ < _____ < _____

2. (*18 points*) Solve each of the following integrals. Show your work clearly and put your final answers in the boxes provided.

a. (*6 pts*) $\int \ln x \, dx$ (Show your work; don't just use the formula.)

b. (*6 pts*) $\int 2\pi x(8 - x^{\frac{3}{2}}) \, dx$

c. (*6 pts*) $\int \sec^4 x \tan^4 x \, dx$

3. (*18 points*) Solve each of the following integrals. Show your work clearly and put your final answers in the boxes provided.

a. (*6 pts*) $\displaystyle\int_2^e (x \ln x)^{-1}\, dx$

b. (*6 pts*) $\displaystyle\int \left(1 + \frac{1}{e^x}\right)^2 dx$

c. (*6 pts*) $\displaystyle\int \frac{1}{(x^2+4)^{\frac{3}{2}}}\, dx$

4. (*20 points*) For each problem below, express the quantity described in terms of definite integrals. Use the boxes provided, and show work and/or graphs to earn partial credit. Your answers should *not* involve the letters f and/or g. **DO NOT SOLVE THE INTEGRALS!**

a. (*4 pts*) The signed area between the graph of $f(x) = 9 - x^2$ and the x-axis on $[0, 4]$.

b. (*4 pts*) The "true" area between the graph of $f(x) = 9 - x^2$ and the x-axis on $[0, 4]$.

c. (*4 pts*) The area between the graphs of $f(x) = 9 - x^2$ and $g(x) = 7 - x$ on $[1, 5]$.

d. (*4 pts*) The average value of the function $f(x) = 9 - x^2$ on $[-3, 3]$.

e. (*4 pts*) The arc length of the graph of $f(x) = 9 - x^2$ on $[-2, 1]$.

5. (*18 points*) For each problem below, express the quantity described in terms of definite integrals. Use the boxes provided, and show work and/or graphs to earn partial credit. You answers should *not* involve the letter f. **DO NOT SOLVE THE INTEGRALS!**

a. (*6 pts*) The volume of the solid of revolution obtained by rotating the region between the graph of $f(x) = x^2$ and the x-axis from $x = 0$ to $x = 2$ around the y-axis, using discs and/or washers.

b. (*6 pts*) The volume of the solid of revolution obtained by rotating the region between the graph of $f(x) = x^2$ and the x-axis from $x = 0$ to $x = 2$ around the line $x = 3$, using shells.

c. (*6 pts*) The work required to pump all of the water out of the top of a cylindrical tank that has a radius of 2 feet and a height of 5 feet. (Hint: The work W involved in lifting an object with weight F through a distance d is $W = Fd$, and the weight of water is 62.4 pounds per cubic foot.)

Survey Questions: (worth 2 extra credit points)

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SCRAP WORK