# TEST I

Math 232

February 11, 2003

**Name:** _____

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).

- Make sure that you follow the directions in each problem and that your answer matches what is asked for.

- Justify your answers algebraically whenever possible. For most problems, work done by calculator will <u>not</u> receive any points (although you may use your calculator to check your answers).

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has seven problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!

- Good luck!

**1.** (*10 pts*) Determine whether each of the following statements are true or false.

    **a.** (*2 pts*)  **T**  **F**   Every exponential function has a horizontal asymptote at $y = 0$.

    **b.** (*2 pts*)  **T**  **F**   The quotient of two algebraic functions is itself an algebraic function.

    **c.** (*2 pts*)  **T**  **F**   For any positive real number $x$ we have $\ln(3e^x) = 3x$.

    **d.** (*2 pts*)  **T**  **F**   If $f(u(v(x)))$ is the composition of three functions, then its derivative is given by $\frac{df}{dx} = \frac{df}{du}\frac{du}{dv}\frac{dv}{dx}$.

    **e.** (*2 pts*)  **T**  **F**   If $0 < k < 1$ then $f(x) = e^{kx}$ is an exponential decay function.

**2.** (*20 pts*) Fill in the blanks or give short answers, as appropriate.

    **a.** (*4 pts*)  $f(x)$ is an exponential function if and only if:

                 _____ is proportional to _____ .

    **b.** (*4 pts*)  $f(x)$ is an exponential function if and only if:

              $f(x)$ has a constant _____ .

    **c.** (*4 pts*)  The function $f(x) = 3e^{x+2}$ can be written in the form $f(x) = Ab^x$ with:

              $A = $ _____ ,   $b = $ _____ .

    **d.** (*4 pts*)  The natural exponential function is continuous at zero. Write this statement as a limit.

    **e.** (*4 pts*)  Complete the following statement using limits. A function $f(x)$ has a vertical tangent line at $x = c$ if:

**3.** (*24 pts*) Calculate the following limits. Show all work so I can see how you arrived at your answer, and circle your final answers. (No calculators except to check your answers.)

**a.** (*6 pts*) $\displaystyle \lim_{x \to \infty} \frac{2}{4 + e^{-2x}}$

**b.** (*6 pts*) $\displaystyle \lim_{x \to 0} \frac{x^{\frac{7}{2}} - x^{\frac{8}{3}}}{x^2}$

**c.** (*6 pts*) $\displaystyle \lim_{x \to 3} \frac{2^x - 8}{3 - x}$

**d.** (*6 pts*) $\displaystyle \lim_{x \to \infty} \frac{3^x}{5 + 7(2^x)}$

**4.** (*8 pts*) Prove that if $k < 0$, then the graph of $f(x) = e^{kx}$ is always decreasing and concave up. (Hint: Use derivatives somehow. Also, remember you are writing a *proof* here, so make sure that your argument is very clear.)

**5.** (*12 pts*) Suppose a quantity $Q(t)$, with $t$ measured in years, increases by 4% each year.

**a.** (*6 pts*) Find a formula for $Q(t)$.

**b.** (*6 pts*) What is the *exact* doubling time of the quantity $Q(t)$?

**6.** (*8 pts*) If $f(x) = x\,e^x$, construct number lines for $f$, $f'$, and $f''$. (Your number lines should be marked at the zeros and DNE points of each function, and should have $+$ or $-$ marked in each subinterval.)

**7.** (*18 pts*) Find each of the following derivatives. Circle your final answers.

**a.** (*6 pts*) If $f(x) = \dfrac{3x + 1}{\sqrt{x^3 - 27x + 4}}$, find $f'(x)$.

**b.** (*6 pts*) If $f(x) = 2^{x^3 + 4}$, find $f'(1)$.

**c.** (*6 pts*) If $y = y(x)$ is a function of $x$, and $3y^2 = 4 + xy$, find $\dfrac{dy}{dx}$.

**Survey Questions:** *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

**SPACE FOR SCRAP WORK**