TEST II

Math 232
March 25, 2003

Name: _____

By writing my name I swear by the honor code.

Read all of the following information before starting the exam:

- Show all work, clearly and in order. You will not get full credit if I cannot see how you arrived at your answer (even if your final answer is correct).
- Make sure that you follow the directions in each problem and that your answer matches what is asked for.
- Justify your answers algebraically whenever possible. For most problems, work done by calculator will not receive any points (although you may use your calculator to check your answers).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, plus some extra credit at the end. Make sure that you have all of the pages!
- Good luck!

1. (*8 pts*) Determine whether each of the following statements are true (T) or false (F).

a. (*2 pts*) **T** **F** $\dfrac{\log_7 a}{\log_7 b} = \log_7 a - \log_7 b.$

b. (*2 pts*) **T** **F** $1 - \csc^2\theta = \cot^2\theta.$

c. (*2 pts*) **T** **F** $\left(\sum_{k=1}^{n} \dfrac{1}{k+1}\right)\left(\sum_{k=1}^{n} k^5\right) = \sum_{k=1}^{n} \dfrac{k^5}{k+1}$

d. (*2 pts*) **T** **F** There are exactly two angles whose sine is $-\frac{1}{4}$.

2. (*18 pts*) Fill in the blanks. (Hint: The first three blanks should be filled in with numbers.)

a. (*3 pts*) $\sin^{-1}(\sin(\frac{3\pi}{4})) =$ _____ .

b. (*3 pts*) $\displaystyle\sum_{k=1}^{1000} \frac{k}{n^2} =$ _____ .

c. (*3 pts*) If $\displaystyle\sum_{k=1}^{4} a_k = 7$, $\displaystyle\sum_{k=0}^{4} b_k = 10$, and $a_0 = 2$, then $\displaystyle\sum_{k=0}^{4}(2a_k + 3b_k) =$ _____ .

d. (*3 pts*) The domain of $f(x) = \sqrt{\ln x}$ is: _____ .

e. (*3 pts*) The domain of $f(x) = \sec x$ is: _____ .

f. (*3 pts*) The domain of $f(x) = \sin^{-1} x$ is: _____ .

3. (*24 pts*) Find each of the following limits. Show all work. (Hint: The last limit requires quite a bit of work.)

a. (*8 pts*) $\lim_{x\to 0} \dfrac{3x}{\sin 2x}$ (do this *without* using L'Hôpital's rule)

b. (*8 pts*) $\lim_{x\to \infty} \dfrac{\cos(\tan^{-1} x)}{x}$

c. (*8 pts*) $\lim_{x\to 0^+} (\csc x)^x$

4. (*8 pts*) Use the fact that $\sin x$ is an even function and $\cos x$ is an odd function to prove that $f(x) = \cot x$ is an odd function.

5. (*8 pts*) Find the coordinates (x, y) where the terminal edge of $\theta = -\frac{5\pi}{6}$ meets the unit circle. Your work should include a picture of the angle in the unit circle and a triangle with labelled sides.

6. (*8 pts*) Approximate the area between the graph of $f(x) = x^2$ and the x-axis from $x = 3$ to $x = 5$ using four rectangles and the Midpoint Sum. Show your work clearly. You will get partial credit for a correct picture of the rectangles you are using. (Hint: Fancy notation is <u>*not*</u> necessary here.)

7. (*24 pts*) Find the derivatives of the following functions. Show any relevant work. (Hint: The last derivative requires more work than the first two.)

a. (*8 pts*) $f(x) = \csc^2(3^x)$

b. (*8 pts*) $f(x) = \ln|\sin^{-1}(x^2)|$

c. (*8 pts*) $f(x) = (\ln x)^x$

8. (*2 pts*) Write whatever you like in this box:

Survey Questions: *(2 extra credit points)*

Name a question or topic that could have been on this test, but wasn't.

How do you think you did?

SPACE FOR SCRAP WORK